

The Spider in the Hedge

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Abstract

This paper provides an empirical study of the effectiveness of hedging the spider, a passive exchange traded fund (ETF) that replicates the S&P500 index. The spider is by far the largest ETF in the world: trading on the spider has grown so much during the past few years that it is now amongst the few most traded securities in the AMEX. The large net daily creation and redemption orders of recent years pose a problem to the market makers in the spider, as the orders may be too large to execute in the cash market. They face a decision about whether to hedge spider positions on their own book; and if so, how should they hedge? We have employed several sophisticated minimum variance estimates for the future hedge ratio, including OLS regression, an ECM to account for maturity effects and the cointegration of the spot and the future prices and, to the ECM residuals we apply EWMA and number of bivariate GARCH models to account for time-variation in the hedge ratio. We have applied these models to daily data for a 1-day rebalancing frequency and to weekly data for a 5-day re-balancing frequency, using data since the spider's inception until the end of 2004. Marginal differences in the 'optimal' hedge ratios are apparent, but they are simply too small to have any significant effect on the hedged portfolio volatility. In out-of-sample testing we find that the naïve hedge where an equal and opposite position is taken in the future performs as well as the more technically sophisticated models, at both the daily and the weekly re-balancing frequency. Finally, we have considered the differences between hedging the spot index and hedging the spider. The efficiency of hedging the spider is superior to that of the index and the spider hedged portfolios have significantly lower volatility than the spot index hedged portfolios.

Acknowledgement: Many thanks to our esteemed colleague at the ISMA Centre, Prof. Charles Sutcliffe for providing useful comments on an earlier draft of this paper and for agreeing with our conclusions. Any errors or omissions are our own.

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I Introduction

In 1993 the American Stock Exchange (AMEX) introduced an exchange-traded instrument for investment in the S&P500 index, the Standard and Poor's Depository Receipt (SPDR) commonly called the 'spider'. By December 2004 it had become the most widely traded Exchange Traded Fund (ETF) in the world, having 54.83bn US\$ under management and representing nearly one quarter of the total market in passive (i.e. index tracking) ETFs in the US. Figure 1 depicts the evolution of the daily traded volume of spider shares as percentage of the NAV of the fund and Table 1 summarises this by the annual average traded volume as a percentage of the outstanding shares. Since 2002 the average daily traded volume has been over 10% of the total outstanding shares. Table 2 shows the number of spider shares outstanding at the end of each year and the annual growth rates. The average annual growth rate was 46% over the last eleven years. The number of spiders increased to approximately 461,947,000 by the end of 2004. The increasing traded volume combined with the increasing number of outstanding shares has resulted in spiders being one of the most traded securities at AMEX.

ETFs offer investors many benefits, including relatively low trading costs and management fees, diversification, tax efficiency, and liquidity. Investors also have all the benefits of exchange trading such as short selling and limit orders. Not surprisingly therefore, the market in passive ETFs has grown very rapidly since the inception of the spider. According to the Investment Company Institute 2004 Mutual Fund Fact Book, the assets under management by passive ETFs in the US have grown by an average of 85% per annum during the last 10 years, compared with only 15% average annual growth of assets under management in the mutual fund industry as a whole. By December 2004 there were over 150 different passive ETFs listed on US exchanges.

Most academic research on ETFs has focused on the spider but, in contrast to our research, it has concerned the microstructure effects of its introduction. Several studies have shown that it has improved the pricing efficiency in futures and options markets (Akhert and Tian, 2001; Switzer, Varson, and Zghidi, 2000; Chu and Hsieh, 2002). Other academic research has examined the price characteristics of the spider (Akhert and Tian, 2000) the reasons for its underperformance relative to the index (Elton *et al*, 2001) and its tax advantages relative to index funds (Poterba and Shoven, 2002). Chu, Hsieh and Tse (1999) show that the S&P500 futures market still provides the dominant price discovery function, with prices of the spider as well as the S&P500 index adjusting very rapidly.

Despite the numerous studies on hedging the S&P500 index with the future, this paper represents the first study (of which we are aware) of hedging the spider with the future. But just as the S&P500 became the natural testing ground for minimum variance hedging research in the 1980's

and 1990's, now the spider is the more obvious choice. Apart from the fact that this is a real problem for the market makers and for investors wishing to take market neutral portfolios, the spider closes trading at the same time as the future so daily closing price data are synchronous, which is not the case for the cash market in stocks. The remainder of the paper is structured as follows: Section II begins by examining the perspective of the market makers in the spider, who have the choice to either close open positions in the spider by going to the cash market and then creating or redeeming spider units with the Trustees, or hedging their position using the S&P500 future. If they decide to hedge they face another decision about the model used to compute the optimal hedge ratio and the time horizon over which they re-balance the hedge. Section III reviews the research on optimal futures hedging and the empirical findings for the S&P500 index and section IV describes the theoretical basis for some common statistical models for estimating the minimum variance hedge ratio. Section V describes the data used in this study and presents our empirical results. For the particular problem facing the market makers in the spider, we compare the effectiveness of some commonly used statistical models for optimal hedging, considering re-balancing the futures hedge at both the daily and the weekly frequency. Section VII summarizes our results and concludes.

II Why Hedge the Spider?

The SPDR Trust is a unit investment trust designed to correspond to the price and yield performance of the S&P500 Index. It issues and redeems shares only in large lot sizes, multiples of 50,000, called 'creation units'. The SPDR Trust continually issues and redeems 'in-kind' shares based on the most recently calculated net asset value (NAV) of the creation units. Each day the Trustee discloses the securities composition of the portfolio, reflecting the relative weighting of the current S&P500, and a cash component, related to dividend payments and other balancing amounts. The NAV per share is determined by the total value of the portfolio and other assets minus all liabilities (including accrued expenses and dividends payable) and divided by the total number of outstanding spiders. The NAV per creation unit is computed by multiplying the NAV per share by 50,000. The Trustee calculates the NAV at the closing time of the regular trading session on the New York Stock Exchange, Inc., i.e. usually 4:00 p.m. EST. During the trading day every 15 seconds the Sponsor, PDR Services LLC, discloses to the Exchange the sum of the dividend equivalent payment plus the current value of the stock portion of the portfolio. Hence the spider may be transacted at market price at any time during the trading day.

The spider delivers to the redeeming shareholder low cost securities in-kind so taxable capital gains are relatively low. Aside from the tax advantages, the in-kind redemption and creation of spider shares allows arbitrage between the S&P500 stocks and the spider shares. With intra-day trading there is no guarantee that the price of the spider at the end of the day will correspond to its end of

day NAV. However, financial intermediaries – normally the market makers in the fund – maintain a no-arbitrage relationship between the spider and the underlying portfolio. The market makers can create or redeem units,¹ so that if the spider price rises too far above the most recently calculated NAV it will pay the market maker to buy stocks to create new units and if the spider price falls too far below the NAV it will pay the market maker to redeem units of the spider for the constituent stocks. This arbitrage mechanism ensures that the market price of the spider does not deviate too far from the most recently calculated NAV. As a result, any end of day ‘mispricing’ of the spider relative to its end of day NAV will normally be arbitrated away during the first few hours of trading on the next day.

At the end of a day the net creation and redemption of spider shares can be very significant, as shown in figure 2. By the end of 2004, with almost 462 million shares outstanding, a net creation or redemption of 1% of the total fund value represented transactions amounting to over half a billion USD. We can see from the figure that a net daily creation or redemption of 1% of the outstanding shares is quite typical. Often much larger amounts are in net demand or supply. For instance, on 1st July 2003 there was a net redemption of over 29.5 million spider shares, with a face value of nearly 2.9 billion USD and representing 6.7% of the total number of spider shares outstanding.

Very large net creation or redemptions are not uncommon. Sometimes they occur on or about a dividend date when trading volume increases markedly with investors trading for their relative tax advantages. For instance, on 15th December 2003 there was a net creation demand for over 54 million spider shares, with a face value of nearly 6 billion USD and representing 15% of the total number of spider shares outstanding. After the dividend date on 22nd December 2003 there followed a net redemption demand for 22.65 million shares, with a face value of nearly 2.5 billion USD and representing more than 5% of the outstanding shares.

Clearly when matching creation and redemption orders at the end of a day a market maker could have a very large net creation or redemption which may be too large for trading in the cash market. The market maker may convert only some, or even none, of the net demand or supply into buying or selling S&P500 stocks or possibly trading spiders with a competitor. The market maker may choose to take part or all of the net demand or supply onto his own account, having a large long or short position in the spider until the following day, when there could be a net creation/redemption that offsets this position. But then, the market maker faces another decision whether to hedge the risk of the spider and if so, how?

¹ Market makers can only create and redeem shares at the end of the day using the daily NAV of creation units. The order can be giving at any time but it is based on the daily NAV, which does not change throughout the day for the purposes of creating/redeeming shares. The 15 seconds disclosure is only informative.

III Hedging the S&P500 with Futures

Over 70% of trading of the new American-style spider options is by the market makers.² But this is more likely to be the provision of liquidity to speculators than to spider market makers wishing to hedge their own positions. The problem with hedging with options is the additional uncertainty induced by the unknown volatility processes. Trading options is, anyway, more expensive than trading futures. For these reasons the market makers in the spider are much more likely to hedge with the index future.

Several academic studies show that the introduction of the spider has enhanced the efficiency of futures markets. For instance, Switzer, Varson, and Zghidi (2000) and Chu and Hsieh (2002) examine the possibilities for S&P500 futures arbitrage before and after the introduction of the spider.³ The spider has the effect of reducing the no-arbitrage range for the future with the most significant change being to the lower boundary. Arbitrage costs are reduced significantly when the spider is traded in place of the constituent stocks of the S&P500 index. In particular the short selling of the S&P500 index is facilitated using the spider, which being a basket security is exempt from the up-tick rule that prevents short selling except after an up-tick. Short selling is also much less expensive using the spider instead of index replication. Hence the frequency and length of no-arbitrage boundary violations, and lower boundary violations in particular, have declined since the spider began trading.

As trading in the spider grows so the future's correlations with both the index and the spider increases. For several years now even conditional correlation estimates have stayed very close to unity (see, for instance, figure 12). Couple this with the numerous cost and liquidity advantages of trading in futures and we see that the S&P500 future is the ideal hedging instrument for the spider.

We shall be examining which is the optimal of various futures hedge ratios available to the market makers that wish to hedge long or short positions in the spider over night, or over a few days. The benchmark for performance measurement is the simple 1:1 hedge where every unit of the spider is hedged with the equivalent units of futures (in the ratio 10:1), and the futures contract size is \$250 times the S&P500 stock price index. Against this we assess the effectiveness of implementing minimum variance hedge ratios that are computed according to a variety of different assumptions, including the more technically advanced assumptions that have been the subject of some interesting recent research.

² The Options Clearing Corporation at www.theocc.com

³ Akhert and Tian (2001) examine the effect of the introduction of the spider on the efficiency of the index options market.

As the S&P500 index represents a very large and liquid universe of stocks it is not surprising that it has been used as an empirical testing ground for research on optimal hedging of stock indices with futures. Sutcliffe (2005) provides an extensive survey of the academic literature in this area, including numerous studies of hedging the S&P500 index portfolio with the future where the focus has been on examining differences in optimal hedge ratios due to the hedging horizon, the frequency of rebalancing and the in-sample period and frequency of the data used. The data have mostly been at the daily frequency, and these have often been taken as S&P500 index and future closing prices (Figlewski, 1985; Junkus and Lee, 1985; Peters, 1986; Graham and Jennings, 1987; Merrick, 1988; Kolb and Okunev, 1992; Bera, Bubnys and Park, 1993; Stoll and Whaley, 1993; Ghosh, 1993; Hancock and Weise, 1994; Lien, Tse & Tsui, 2002 and others). Since futures' trading ends 15 minutes after trading in the underlying cash portfolio, much of the empirical analysis just cited actually used non-synchronous data. However some studies of the S&P500 are at the weekly or monthly frequency (Hill and Schneeweis, 1984; Lindahl, 1991, 1992; Lien & Luo, 1993; Park and Switzer, 1995; Geppert, 1995 and Miffre, 2004) or at the transactions level (e.g. Benet & Luft, 1995).

Early work on weekly data during 1980's produced some surprisingly low optimal hedge ratios, albeit with low efficiency. Hedge ratios based on daily data were generally lower than those based on weekly data, although Hancock & Weise (1994) analyzed daily data on the S&P500 in the late 1980s finding hedge ratios that averaged about 0.95, which is similar to the more recent studies on S&P500 weekly data. Not surprisingly there is evidence to suggest that optimal hedge ratios increase towards unity as the horizon of the hedge tends towards the maturity date of the futures contract (Lee, Bubnys and Lin, 1987). The maturity effect is clearly important to capture, and studies that do so have reported higher optimal hedge ratios. Amongst others, Lien and Luo (1993) and Ghosh (1993) argue that, because the basis risk converges towards zero as the future approaches expiry, it is also important to include the effect of cointegration between the stock index and futures when estimating optimal hedge ratio. At least one should account for the time to expiry of the futures contract at the time when the hedge is lifted. Lien & Luo (1993) and Ghosh (1993) introduced an error correction term in the hedge ratio regression that can capture the maturity effect as spot and futures prices converge at the contract maturity. This produced average hedge ratios of 0.89 in both studies, based on daily S&P500 data.

Beyond accounting for cointegration between the spot and future, there has been a considerable amount of work recently on the use of generalized autoregressive conditional heteroscedasticity (GARCH) models for gaining more precise estimates of short-term optimal hedge ratios for hedging a stock index with the future over a period of between one day and one week. Checcetti, Cumby and Figlewski (1988) argue that the standard technique of regressing spot returns on futures returns and taking the optimal hedge ratio to be the estimated slope coefficient is unsatisfactory. An

ordinary least squares (OLS) regression of spot returns on future returns assumes that the optimal hedge ratio is constant over time. But in this case the maturity of the hedge should be irrelevant, even though if the hedge is lifted at the expiry of the future, the basis is zero and the optimal hedge ratio must be unity.

Checchetti, Cumby and Figlewski (1988) claimed that time-varying hedge ratios estimated using a simple GARCH model are more effective than the constant hedge ratios estimated via OLS. Subsequently, Baillie and Myers (1991), Kroner and Sultan (1991), Park and Switzer (1995), (Lien, Tse & Tsui, 2002), Brooks, Henry and Persaud (2002), Miffre (2004) and many others have applied bivariate GARCH models to estimate optimal hedge ratios.

However, very few if any of these studies provide a convincing argument in favour of using GARCH minimum variance hedge ratios for stock indices. There is no conclusive evidence that this can improve upon the performance of OLS or naïve 1:1 hedging strategies. Commodities hedges, of course, are likely to be quite different as the basis risk is so much larger. But during the last five years the conditional correlation between the future and both the S&P 500 index and the spider, has remained close to one (see figure 12). Hence the volatilities are likely to be similar and the minimum variance hedge ratio should be very close to one. Several studies that claim superiority of bivariate GARCH for stock index hedging with daily re-balancing do not use synchronous data and the estimated hedge ratios are lower than they should be.

Lien (2005) criticizes some findings that technically sophisticated models produce no better hedging than the OLS hedge arguing that these studies use inappropriate, in-sample, effectiveness measures. We shall use a simple effectiveness measure, the proportion of variance reduced by the hedge, proposed by Ederington (1979) but estimated out-of-sample as a time varying metric. Nevertheless it should be noted that Lien (2005) even questions the appropriateness of out-of-sample effectiveness measures.

It came as somewhat of a surprise to us, being such enthusiasts for time series analysis and having devoted much time extolling the virtues of cointegration and GARCH,⁴ that the ECM and GARCH refinements to computing minimum variance hedge ratios for a stock index are simply 'not worth the biscuit'. However, we are not the only authors to have reached this conclusion (see Sutcliffe (2005) Chapter 9). To demonstrate this, and explain why it is so, is one of the aims of this paper.

⁴ See www.ismacentre.rdg.ac.uk/alexander

IV Minimum Variance Hedge Ratios

Consider a T -maturity future that at some time t is used to hedge a spot position with price S_t . Assuming constant risk-free T -maturity interest rate r and dividend yield q , the theoretical or ‘fair’ value for the future is

$$F_t^* = \exp((r - q)(T - t))S_t \quad (1)$$

and the market price is

$$F_t = F_t^* + x_t S_t \quad (2)$$

where x_t has been termed the ‘mispricing’ of the market price of the future compared with the fair value. But of course it is the spot, or fair value of the future rather than the market value of the future that is ‘mispriced’. We know that in liquid markets such mispricing is very temporary as, typically, the spot price will adjust very quickly to changes in the market price of the future, so that the fair price of the future rapidly moves back towards to its market price. The market price of the future responds first to market news because the futures are more liquid.⁵

Now consider a cash position in the index at time t that is hedged by selling $\beta_t(\tau)$ units of a T -maturity future with market price F_t assuming the position will be closed at time $t + \tau$, with $0 < \tau < T$. Following Figlewski (1984) we base the optimal hedge ratio on the τ -period index return:

$$R_t^S(\tau) = \frac{(S_{t+\tau} - S_t + D_t(\tau))}{S_t} \quad (3)$$

and the futures ‘return’, defined as:

$$R_t^F(\tau) = \frac{(F_{t+\tau} - F_t)}{S_t} \quad (4)$$

where $D_t(\tau)$ represents the present value of dividends received between time t and time $t + \tau$.

Denote the variance of $R_t^F(\tau)$ at time t by $\sigma_{F,t}^2(\tau)$ and the covariance between $R_t^S(\tau)$ and $R_t^F(\tau)$ by $\sigma_{SF,t}(\tau)$. Then the minimum variance hedge ratio for a hedge of duration τ is given by:

$$\beta_t^*(\tau) = \frac{\sigma_{SF,t}(\tau)}{\sigma_{F,t}^2(\tau)} \quad (5)$$

With this hedge ratio the return on the hedged portfolio between time t and time $t + \tau$ is $R_t^S(\tau) - \beta_t^*(\tau)R_t^F(\tau)$ and the variance of this return is $\sigma_t^*(\tau) = \sigma_{S,t}^2(\tau)(1 - \rho_{SF,t}^2(\tau))$, where $\sigma_{S,t}^2(\tau)$ and $\rho_{SF,t}(\tau)$ denote the variance of the τ -period index return and the correlation between the τ -period returns on the index and the future at time t .

⁵ Chu, Hsieh and Tse (1999) show that the future still plays the dominant role in the price discovery process, even after the introduction of the spider.

The simplest of all the hedge ratios considered in this study – apart from the so-called ‘naïve’ 1:1 ratio – is the minimum variance hedge ratio (5) estimated using OLS. We use a rolling in-sample estimation periods of (a) six months and (b) one year and consider the cases $\tau = 1$ and 5 using first daily and then weekly (Friday close) data. The R^2 from each regression is a standard (but in-sample) measure of ‘efficiency’ of the hedge.

In employing OLS one faces the ambiguity of using a constant parameter model to estimate a parameter value that is not necessarily 1, although we know that the parameter must be 1 at some point in time (i.e. when the future expires). This is one reason why we are interested in time-varying parameter models that can also account for the fact the spot and futures are tied together and hence adjust the parameter towards 1 as the future approaches expiry.

To model the effect of spot-futures cointegration, i.e. that the spot and future prices are ‘tied together’ we shall include the carry cost in a bivariate error correction model (ECM) for deriving the optimal futures hedge ratio. To see why, take logarithms of (1), giving:

$$\ln F_t^* - \ln S_t = (r - q)(T - t)$$

This shows that if the carry cost, $cc_t = (r - q)(T - t)$ is stationary the logarithm of the spot price and the logarithm of the fair value of the futures price should be cointegrated with cointegrating vector $(1, -1)$. For a stock index, the spot and futures prices are most certainly cointegrated, although this need not be true for commodities (see Baillie and Myers, 1991). However the carry cost need not be the *most* stationary linear combination of the log of the market price of the future and the log of the spot price. In fact, it may not be stationary at all in commodity markets. Nevertheless if the mispricing of the future relative to its fair value is small it is reasonable to assume the error correction term in the error correction model is equal to the carry cost.⁶ We shall adopt this, more intuitive, formulation and hence specify the following error correction model:

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=1}^n \boldsymbol{\Gamma}_i \mathbf{y}_{t-i} + \boldsymbol{\pi} cc_{t-n} + \boldsymbol{\epsilon}_t \quad (6)$$

where

$\mathbf{y}_t = \begin{pmatrix} r_{F,t}(\tau) \\ r_{S,t}(\tau) \end{pmatrix}$ is the vector of τ -period log returns on the future and spot,

$\boldsymbol{\mu} = \begin{pmatrix} \mu_F \\ \mu_S \end{pmatrix}$, $\boldsymbol{\Gamma}_i = \begin{pmatrix} \Gamma_{i,F,F} & \Gamma_{i,F,S} \\ \Gamma_{i,S,F} & \Gamma_{i,S,S} \end{pmatrix}$ and $\boldsymbol{\pi} = \begin{pmatrix} \pi_F \\ \pi_S \end{pmatrix}$ are constants, and

⁶ The carrying cost for the spider does not include the continuous dividend yield, because the dividend is determined on the ex-dividend date which coincides with the maturity date of the futures contract, i.e. the third Friday in each of March, June, September and December. The beneficial owners registered on the ex-dividend date are entitled to receive the dividends accumulated during the previous quarterly dividend period.

$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{F,t} \\ \varepsilon_{S,t} \end{pmatrix}$ is the vector of unexpected returns to future and spot.

The true risk that needs to be hedged is then the conditional variation in $\boldsymbol{\varepsilon}_t$. We are particularly interested to know whether, as claimed in part of the research cited above, it is more efficient to extend the assumptions (6) to *also* allow for time-variation in the optimal hedge ratio (5). The time-varying optimal hedge ratio between spot and futures prices is given by:

$$\tilde{\beta}_t^*(\tau) = \frac{\tilde{\sigma}_{SF,t}(\tau)}{\tilde{\sigma}_{F,t}^2(\tau)} \quad (7)$$

where $\tilde{\sigma}_{SF,t}(\tau)$ and $\tilde{\sigma}_{F,t}^2(\tau)$ denote the conditional covariance of the unexpected returns to spot and future, and the conditional variance of the unexpected future return respectively.

The third set of minimum variance hedge ratios employs exponentially weighted moving averages (EWMA) to estimate the numerator and denominator of (7). The ECM-EWMA estimates are simple to calculate (compared with the bivariate GARCH estimates considered below). However, they do depend on an ad hoc choice for the value of the smoothing constant. We use two different values for the smoothing constant, of 0.90 and 0.95 respectively.⁷

Beyond the ECM-EWMA, to model time-variation in a fully conditional bivariate GARCH framework, we assume that

$$\boldsymbol{\varepsilon}_t | \boldsymbol{\Omega}_{t-1} \sim N(0, \mathbf{H}_t)$$

where $\boldsymbol{\Omega}_{t-1}$ denotes the information set at time $t-1$ and

$$\mathbf{H}_t = \begin{pmatrix} \tilde{\sigma}_{F,t}^2(\tau) & \tilde{\sigma}_{SF,t}(\tau) \\ \tilde{\sigma}_{SF,t}(\tau) & \tilde{\sigma}_{S,t}^2(\tau) \end{pmatrix} \quad (8)$$

Hence our fourth set of minimum variance hedge ratio estimates combines ECM with GARCH models instead of EWMA. We use a variety of bivariate GARCH(1,1) parameterisations of the dynamics of \mathbf{H}_t , each of which has been well documented. We use a variety of BEKK specifications (Engle and Kroner, 1995) and the dynamic conditional correlation model of Engle, (1999). The BEKK specification ensures positive definiteness while imposing cross equation restrictions (e.g. the scalar BEKK imposes that persistence in volatility and correlation are the same). The t -BEKK replaces the conditional normality assumption with that of conditionally t -distributed error terms. The dynamic conditional correlation (DCC) model is an extension of the constant conditional correlation estimator of Bollerslev (1990) where the correlation matrix has

⁷ The choice of values is somewhat arbitrary here, just as is the choice of in-sample estimation period for OLS regression. We have simply chosen one relatively low value and another relatively high value.

time-varying estimates based on a constrained form of the ‘diagonal vech’ GARCH parameterization.

We measure effectiveness of each hedge using the proportional variance reduction measure proposed by Ederington (1979). Each day (or week) we estimate the hedge ratio, based on a rolling in-sample period, which determines the futures position to be taken at the end of the day (or week) until the following day (or week). The sample is then rolled one period, the hedge ratios re-estimated, and the hedge re-balanced and held until the end of the next day (or week). We thus form an out-of-sample hedge portfolio returns series. Then, denoting by V_H the variance of the hedged portfolio return and V_U the variance of the un-hedged position we obtain an estimate for hedge effectiveness given by:

$$E = \frac{(V_U - V_H)}{V_U} \quad (9)$$

V Results

We use Bloomberg daily closing price data on the spider and the S&P500 nearest future over a twelve-year period, from February 1993 to December 2004. Trading in the spider ceases at 4:15 p.m. EST on AMEX and futures on the S&P500 are traded until 4:15 p.m EST on CME. For comparison with previous studies on S&P500 index hedging we shall also estimate the corresponding hedge ratios for a cash position on the index, using daily closing prices even though trading in the underlying stocks closes at 4:00 p.m. EST. Weekly data are also taken over the same period, using the Friday closing prices.

We shall be drawing a number of comparisons in our results: First we compare each estimate of the hedge ratio for the spider with the corresponding hedge ratio estimate for the index itself. Since trading the index basket of shares is much less expensive with the spider (and short selling in particular is facilitated since spider trading is exempt from the up-tick rule) the ‘mispricing’ of the index future relative to the spot is likely to last longer than mispricing of index future relative to the spider. This should have the effect of decreasing the optimal hedge ratio for hedging a portfolio that replicates the index. Another reason why optimal hedge ratios for the index are likely to be lower than those for the spider is that price movements in the future during the last 15 minutes before close of the market may have the effect of reducing the covariance between the spot index and the future and hence reduce the minimum variance hedge ratio even further. A second type of comparison and perhaps the most interesting, is of the minimum variance hedge ratios for the spider that are obtained from the different models. These are estimated on a rolling basis for the

period January 1994 to December 2004. All bivariate GARCH models were estimated using the Matlab UCSD GARCH toolbox.⁸

Figures 3 to 6 compare the OLS minimum variance hedge ratios for the spider with those estimated for the S&P500 index (Spx), based on daily and weekly data with in-sample estimation periods of 6 months and 1 year. As expected, and for the reasons outlined above, the daily hedge ratios are lower for the index than for the spider, as is a simple measure of the hedge 'effectiveness' given by the in-sample R^2 from the rolling OLS regressions. However the weekly hedge ratios are very similar for the index and the spider, indicating that the last 15 minutes of trading on the future that is captured by the daily data has a significant effect on depressing the estimated hedge ratios for the index. Naturally, the stability of the estimated hedge ratios increases with the length of the in-sample estimation period. What is most notable about these figures is that during the last three years the minimum variance hedge ratios for the spider have been highly efficient and also very close to the 'naïve' 1:1 hedge ratio.

The simple rolling in-sample R^2 shown in figures 4 and 6, are not good measures of performance. OLS is designed to give the lowest in-sample residual variance of all two parameter linear models, so its in-sample R^2 is bound to be high. As Lien (1995) argues, out-of-sample testing is more appropriate especially when some sophisticated hedging models, such as ECM-GARCH, are being examined. Hence as described in the previous section, we match a spot position with an opposite position of beta in the future at the end of each day (or week), we hold this until the end of the following day (or week) and then re-balance. In this way we form a daily (or weekly) out-of-sample returns series. We do not account for transactions costs and this results in a very small positive return on the portfolio in most cases.

Figure 7 graphs the annual volatility of the out-of-sample returns from the OLS minimum variance hedged portfolio and compares this with the annual volatility of the returns on the 1:1 hedged portfolio. All volatilities have been estimated using exponentially weighted moving averages with a smoothing constant of 0.95. For the spider, the volatility of the hedged portfolio is hardly different whether one takes the 1:1 hedge ratio always or attempts to minimize the variance using an OLS estimate of (5). For the S&P500 index small differences between the volatilities of the two portfolios are evident, but only during the 1990's. Since 2000 the OLS hedge has performed more or less identically to the 1:1 hedge for the index as well as for the spider.

The failure of the OLS hedge ratio to improve upon the naïve 1:1 hedge might be attributed to the lack of sophistication of the hedging methodology. We have neither accounted for time to maturity

⁸ Available from http://www.kevinsheppard.com/research/ucsd_garch/ucsd_garch.aspx.

effects in the OLS hedge nor allowed for the assumption that the model's minimum variance hedge ratio is a time-varying parameter. Therefore to address these questions we have estimated minimum variance hedge ratios based on (6) and (7). On a rolling sample window we estimate the ECM model (6) each time applying EWMA and the bivariate GARCH(1,1) models described in the previous section to the residuals. We do not bother to test for spot-futures cointegration as this fact is already very well established in the S&P500 index.

The daily minimum variance hedge ratios for the ECM-EWMA, ECM- t BEKK and ECM-DCC estimates of (7) are shown in figure 8 (for the spider) and figure 9 (for the S&P500 index). There are substantial differences between these estimates and the OLS estimates shown in figure 3. First, the ECM-EWMA, ECM- t BEKK and ECM-DCC estimates have accounted for maturity effects, being based on the ECM residuals. Secondly, variation in the hedge ratio is part of the model, whereas in the OLS case the variation shown in figure 3 can only be attributed to sampling error. As a result, there is substantially more variation in the ECM residual based time varying hedge ratios than in the OLS hedge ratios. We have not included all the different GARCH hedge ratios in these graphs as they were so similar. For instance the scalar BEKK hedge ratios were very close to the t -BEKK ratios.

Based on the out-of-sample returns series constructed for each hedge ratio, table 3 summarizes the average annual volatility of the un-hedged portfolio and the hedged portfolio with these different types of minimum variance hedges. Below the volatility we give the effectiveness measure (9), as an average over the whole sample from 1994-2004. These results show that, for the spider, the minimum variance hedge ratios estimated by different models are not performing any better than the 'naïve' 1:1 ratio. In fact for weekly re-balancing the 1:1 hedge is (marginally) the best. Although for daily re-balancing OLS hedge ratios are the most efficient, on average, the difference is so small that it is unlikely to be significant, or robust to changes in sample. For the S&P500 index, the more sophisticated ECM-EWMA and ECM-GARCH models appear to have some advantage over the 1;1 hedge for daily rebalancing – but again it is marginal, and anyway OLS again gives the highest effectiveness measure, on average.

Since table 3 only presents an average figure for the out-of-sample hedge effectiveness it should be asked whether the technically sophisticated minimum variance hedge ratio models out-perform the 1:1 hedge during certain time periods. Figure 10 shows time-varying estimates of the effectiveness measure (9) for the daily hedges.⁹ We have only included two hedges here: the 1:1 hedge and the ECM-EWMA ratio (with smoothing constant 0.95). We already know from figure 7 that there can be virtually no difference between the effectiveness of the OLS hedge and the 1:1 hedge. We have

⁹ Using a EWMA volatility estimate with smoothing constant = 0.95.

not added the effectiveness measure for the ECM-GARCH models because they display no discernable difference from the time series shown in the figure for the ECM-EWMA. And, for hedging the spider, this is virtually identical to the efficiency of the 1:1 hedge.

At any point in time the variance reduction achieved by tailoring the hedge ratio to minimize the conditional variance of the portfolio offers no discernable improvement on the 'naïve' 1:1 hedge. In the index itself, there are indeed times when the effectiveness of the 1:1 hedge dips below that of the EWMA hedge. However most of these occurrences were in the early part of the sample. In fact, since 2000 there has only been one instance (in July 2004) where the ECM-EWMA hedge was more effective than the 1:1 hedge and this was only very temporary.

Figure 11 shows time-varying estimates of the effectiveness measure (9) for the weekly hedges. Here we have only shown the index hedge – the results for the spider hedge are similar except that all hedges are slightly more efficient. As there is now a more substantial difference between the ECM-GARCH and the ECM-EWMA hedge ratio estimates for weekly hedges, this time we have also included the effectiveness measure for the ECM-DCC hedge. But it is clear from the figure that neither of these models can improve on the efficiency of the simple 1:1 hedge ratio, and the other ECM-GARCH hedge ratios lead to similar conclusions.

Two important facts are evident from figures 10 and 11. The first is that technically sophisticated 'optimal' hedge ratios cannot offer superior hedging effectiveness: naïve 1:1 hedging is perfectly adequate. The second is that hedging the spider is consistently more effective than hedging the index. The key to both these insights lies in the correlation between the future and the cash instrument. Figure 12 shows the daily time-varying (DCC) conditional correlation estimates between the ECM residuals; since the carry cost is included in the explanatory variables for (6), these correlations are estimated after accounting for the maturity effects in the fair value of the future. The future-spider correlation is consistently higher than the future-index correlation, as expected given our comments above. For the last few years the conditional correlation with spider has been very stable indeed, at 0.99 most of the time. The conditional correlation with the index is also stable, but lower, at 0.98.

Investors must see many advantages to including the spider in a hedged portfolio, instead of holding a portfolio of stocks to replicate the index. There are much lower transaction costs, tax advantages, more liquidity and greater ease with taking short positions. Equity market neutral funds may also be interested some simple correlation results. We find that all minimum variance hedged spiders are strongly market neutral. Using the out-of-sample returns for daily and weekly rebalancing we find that the spider in the hedge enhances the market neutrality of the portfolio. The correlation between the S&P 500 index and the portfolio of the spider hedged with the future is

very low and stable. It averages 0.08 over the whole period, for both daily and weekly rebalancing whether we use a 1:1 hedge ratio or one of the more complex methodologies.

However funds that employ technically sophisticated minimum variance hedge ratios for replicating the index with cash positions in stocks do not achieve market neutrality. When hedging a cash portfolio of stocks replicating the index the 1:1 hedge gives a portfolio that also has very low market correlation, averaging less than 0.02 over the period. However the out-of-sample returns to the OLS, ECM-EWMA and ECM-GARCH hedged portfolios for the index portfolio have a relatively high market correlation: over the period 1994 to 2004 it varies between 0.2 and 0.3.¹⁰ Hence, for a traditional fund that invests in a stock portfolio hedged by the future, the 1:1 hedge is strongly market neutral but the other hedge ratios do not give market neutral portfolios.

V Summary and Conclusions

Up to now, research on the spider has focussed on the microstructure effects of its introduction and the subsequently improved efficiency in the S&P500 futures market. Previous empirical studies on the spider have mostly been based on data from the 1990s, but during the last few years the market has expanded and matured considerably. The number of spider shares outstanding more than trebled between the years 2000 and 2004.

This paper examines the problem of hedging the spider with S&P500 futures. Market makers can frequently have very large net creation or redemption orders at the end of a trading day, which may be too large to execute in the cash market. Assuming they choose to hedge spider positions that are left on their book overnight, or over a few days, this paper has addressed the problem of an optimal hedging strategy for the market makers. We use daily and weekly data on the spider since its inception in February 1993 until the end of 2004.

Beginning with a comparison between futures hedging of the spider and futures hedging of the spot S&P500 index, we find that with daily rebalancing the efficiency of hedging is higher for the spider than for the S&P500 spot index. In other words the spider hedged portfolio volatility is lower than the spot index hedged portfolio volatility. On average, daily volatility is reduced from approximately 18.5% for the un-hedged spider to about 3.5% for the hedged spider, and from approximately 17.5% for the un-hedged index to about 4.5% for the hedged index. The increased barriers to arbitrage using the spot index (compared with the spider) and, probably most importantly, the non-synchronous closing times for the spot index and the future in daily close price data may be the reason for this effect. There is much less difference between the hedged spider and the hedge index

¹⁰ There is some variation according to which minimum variance hedge ratio is used. Results available from the authors on request.

portfolio volatilities for weekly volatility – both are around 2.5%, depending on the model used for the hedge ratio. So when the hedge is held for 5 days hedging both portfolios achieves a significant reduction in volatility (but still it is slightly lower for the hedged spider portfolio).

Focussing now on the spider hedge, although the following remarks also hold true for the S&P500 index hedged with the future, we have found no conclusive evidence to suggest that any ‘optimal’ hedge ratio can improve upon the ‘naïve’ 1:1 hedging strategy where an equal and opposite position in the future is taken for the hedge. We advise a 1:1 hedge of the spider with the future, showing that this is as good (if not better) than hedging using more sophisticated statistical models. We have used a time varying out-of-sample methodology to test the effectiveness of many different methods for estimating minimum variance hedge ratios, using: OLS regression; an ECM to account for maturity effects and the cointegration of the spot and the future prices; and the ECM combined with EWMA and several different GARCH models to account for time-variation in the hedge ratio. We have applied these models to daily data for a 1-day rebalancing frequency and to weekly data for a 5-day re-balancing frequency and tested their performance over an eleven-year period. Marginal differences in the ‘optimal’ hedge ratios are apparent, but they are simply too small to have any significant effect on the hedged portfolio volatility. The naïve hedge performs as well as the more technically sophisticated models at the daily frequency and better than them at the weekly frequency. Our results can be attributed to the very high correlation between the spider and the future and thus that their volatilities are also similar.

Whilst there may be interesting research to do on the use of technically sophisticated optimal hedge ratio models for commodity spot and futures, where the spot and the future price can be substantially de-coupled due to the unpredictable nature of the carry cost, for equity indices in general we do not believe that there is a need to employ ‘optimal’ minimum variance hedge ratios. The introduction of the spider has improved the efficiency of the S&P500 futures markets during the last few years and both the spider and the S&P500 index portfolio are more highly correlated than ever before with the future. Minimum variance hedge ratios are now very close to unity for both daily and weekly rebalancing – and would be even higher for less frequent rebalancing than the 1-day and 5-day intervals used in this study. Further research into hedging the spider could be profitably focussed on the optimal re-balancing frequency for the ‘naïve’ 1:1 hedge, using an economic model that balances transactions costs against uncertainty in the hedge.

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Table 1: Average daily volume as a percentage of outstanding shares.

Average Daily Volume	
1993	4.33%
1994	3.86%
1995	2.70%
1996	4.46%
1997	8.52%
1998	10.62%
1999	7.51%
2000	5.62%
2001	6.05%
2002	10.51%
2003	10.28%
2004	11.01%

Table 2: Number of outstanding spider shares at end of year.

	Outstanding Shares	Change %
1993	9900	
1994	9200	-7.07%
1995	16251	76.64%
1996	27108	66.81%
1997	56813	109.58%
1998	98967	74.20%
1999	134670	36.08%
2000	192725	43.11%
2001	264581	37.28%
2002	418241	58.08%
2003	393156	-6.00%
2004	461947	17.50%

Table 3: Summary of average volatilities and effectiveness of minimum variance hedged portfolios

<i>Daily</i>		<i>No Hedge</i>	<i>1:1</i>	<i>OLS (6 mth)</i>	<i>OLS (1 yr)</i>	<i>EWMA ($\lambda=0.9$)</i>	<i>EWMA($\lambda=0.95$)</i>	<i>Diagonal BEKK</i>	<i>Scalar BEKK</i>	<i>t-BEKK</i>	<i>DCC</i>
SPY	Volatility	18.34%	3.63%	3.33%	3.31%	3.69%	3.66%	3.67%	3.69%	3.70%	3.70%
	Effectiveness		96.08%	96.71%	96.73%	95.96%	96.01%	96.01%	95.95%	95.93%	95.94%
SPX	Volatility	17.54%	4.65%	4.29%	4.28%	4.56%	4.50%	4.58%	4.58%	4.59%	4.58%
	Effectiveness		92.99%	94.03%	94.05%	93.25%	93.42%	93.18%	93.18%	93.15%	93.19%
<i>Weekly</i>		<i>No Hedge</i>	<i>1:1</i>	<i>OLS (6 mth)</i>	<i>OLS (1 yr)</i>	<i>EWMA ($\lambda=0.9$)</i>	<i>EWMA($\lambda=0.95$)</i>	<i>Diagonal BEKK</i>	<i>Scalar BEKK</i>	<i>t-BEKK</i>	<i>DCC</i>
SPY	Volatility	17.21%	2.36%	2.38%	2.37%	2.91%	2.83%	2.69%	2.74%	2.87%	3.03%
	Effectiveness		98.13%	98.09%	98.10%	97.13%	97.30%	97.55%	97.47%	97.21%	96.90%
SPX	Volatility	17.13%	2.58%	2.54%	2.54%	2.81%	2.73%	2.72%	2.73%	3.00%	2.93%
	Effectiveness		97.74%	97.80%	97.80%	97.31%	97.46%	97.48%	97.46%	96.93%	97.08%

Figure 1: Daily volume as percentage of outstanding shares

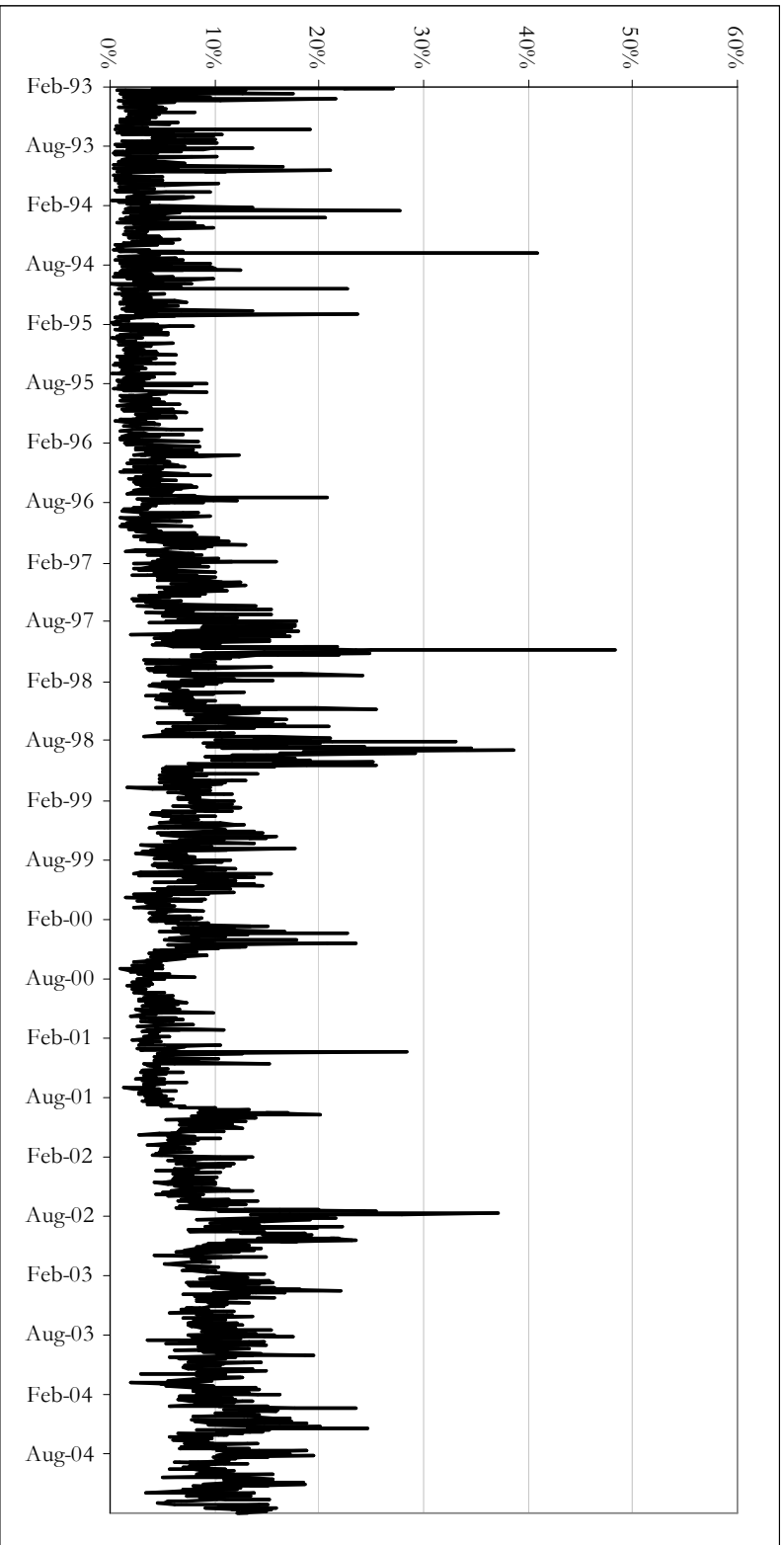


Figure 2: Daily Net Creation/Redemption and Total Number of Outstanding Shares

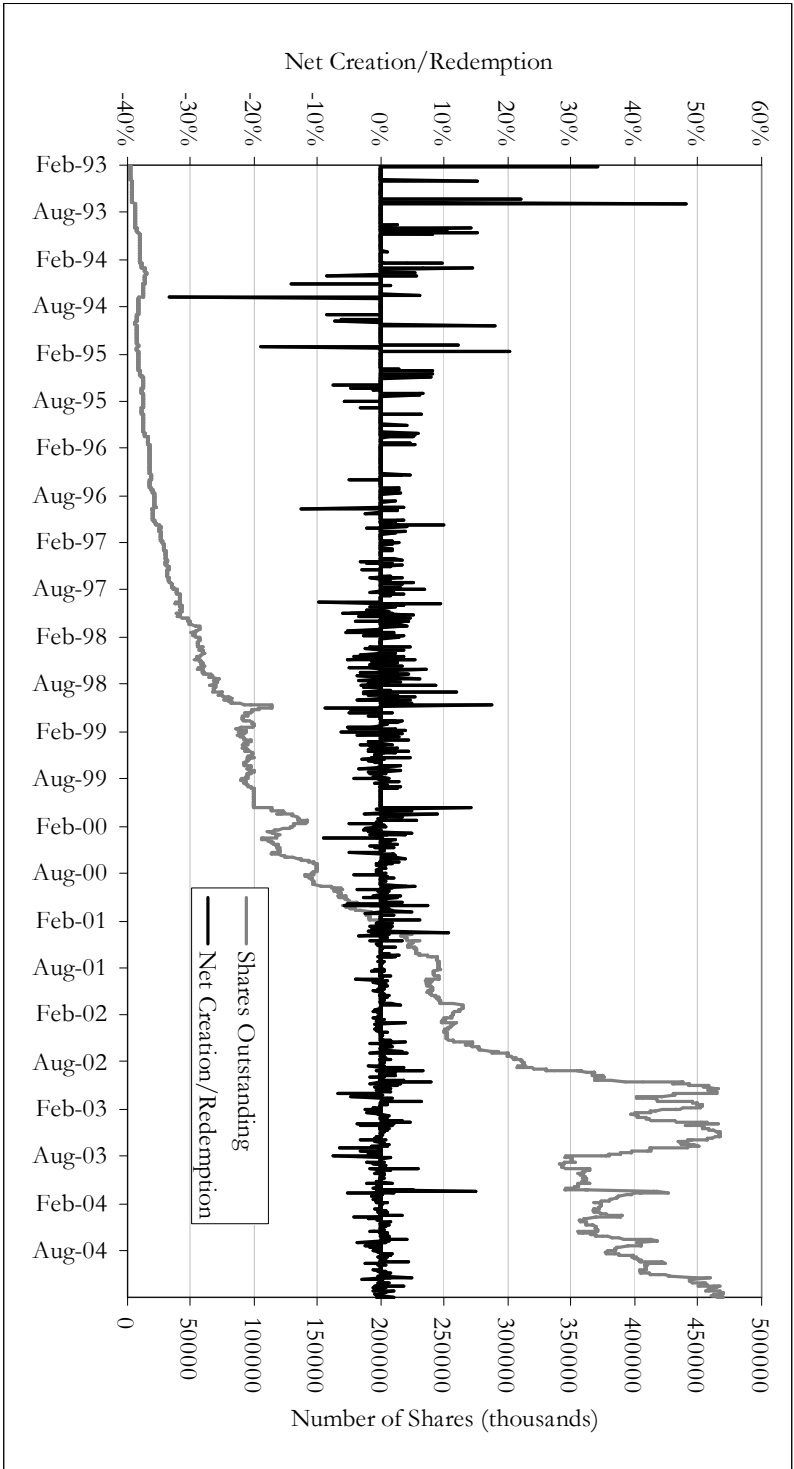


Figure 3: OLS daily hedge ratios

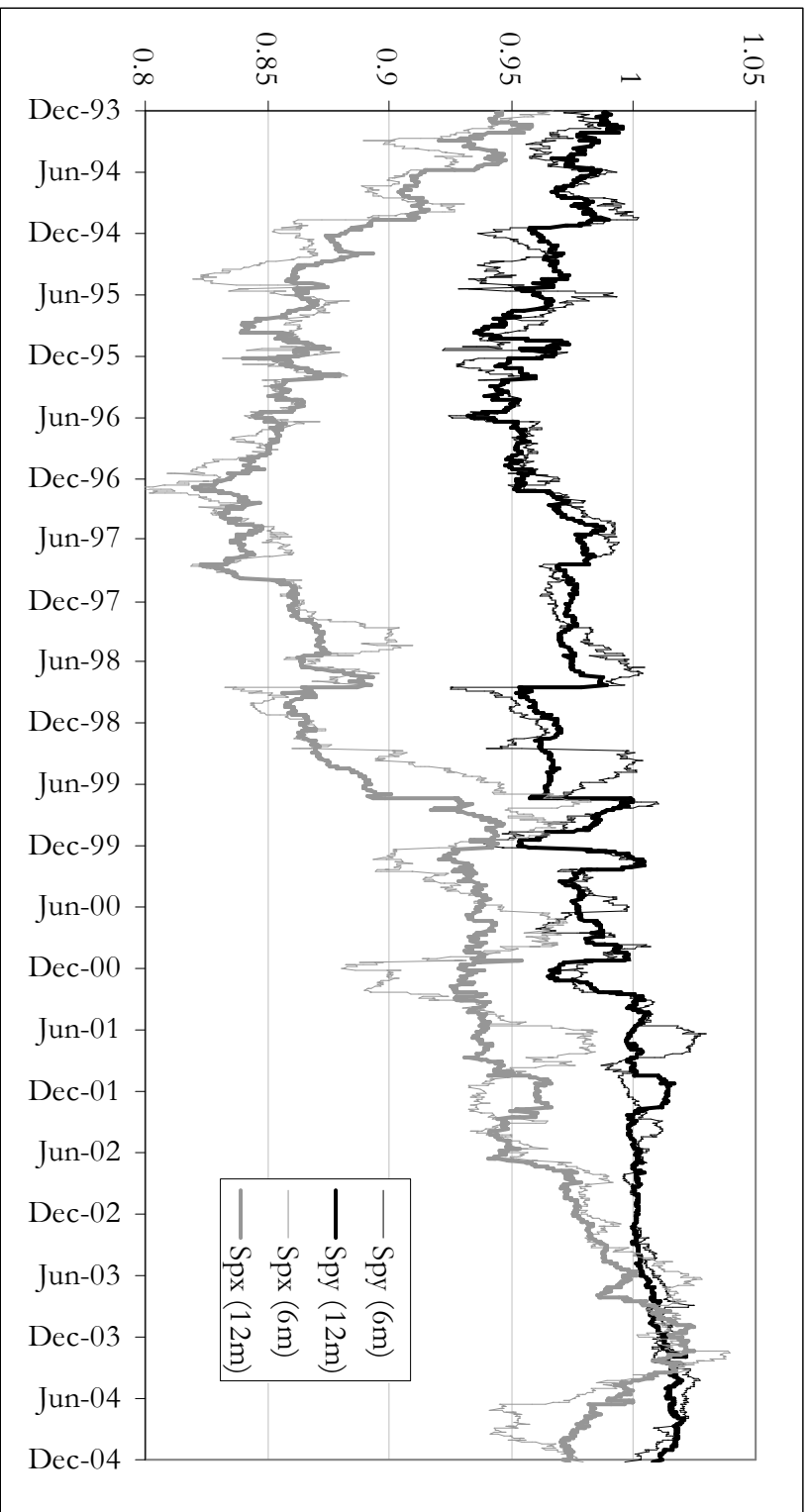


Figure 4: Rolling R^2 of OLS daily hedge ratios

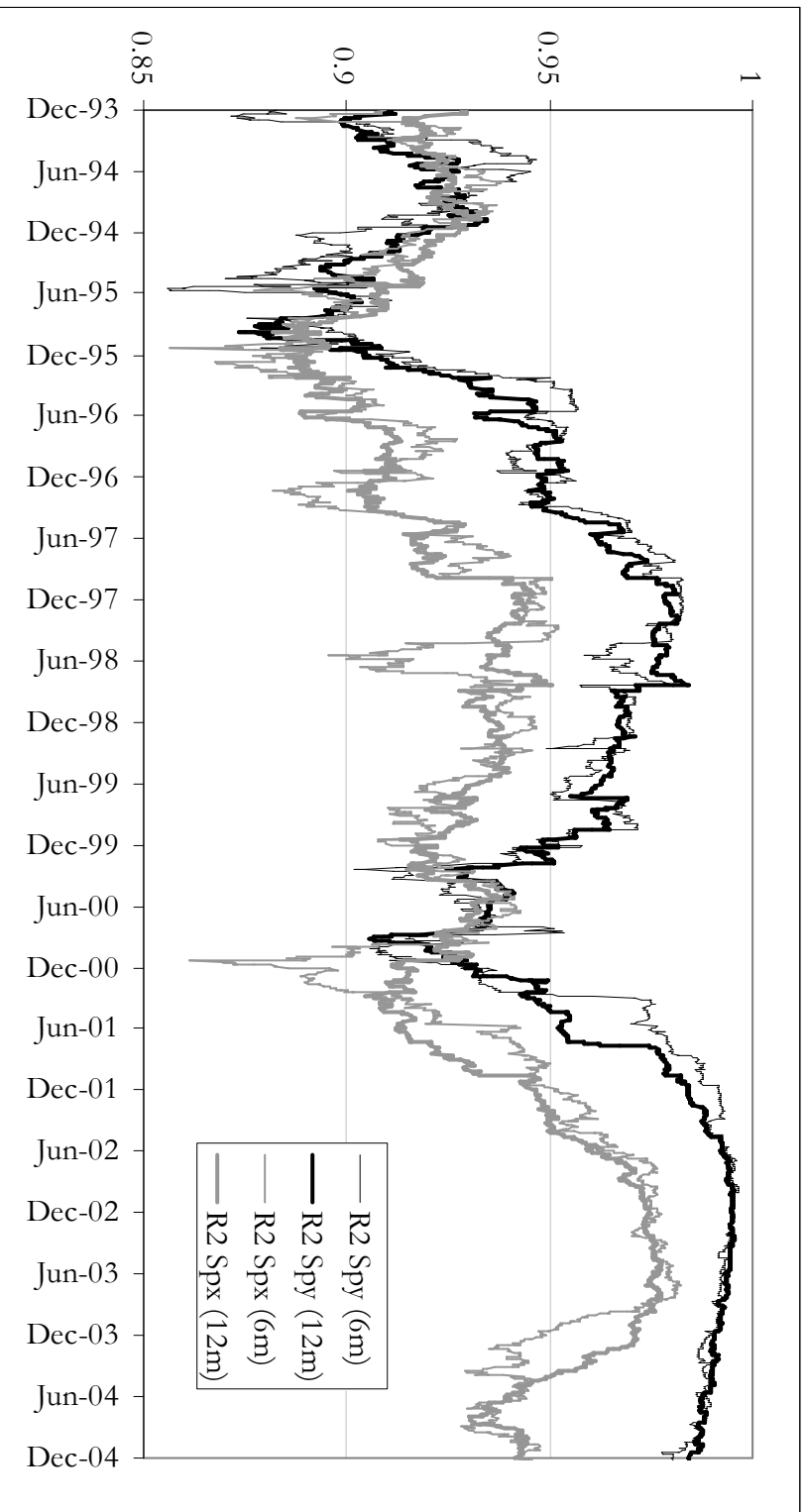


Figure 5: OLS weekly hedge ratios

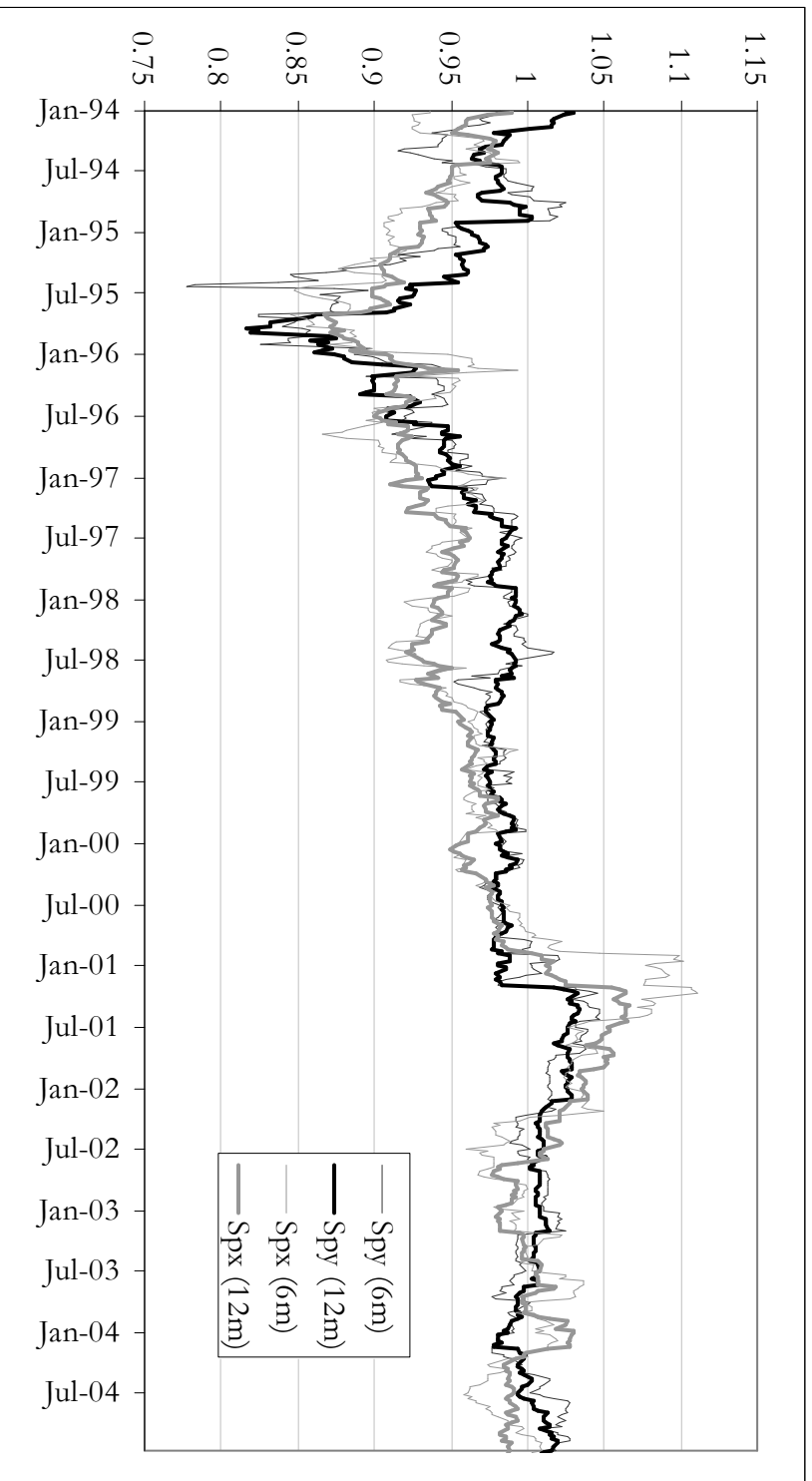


Figure 6: Rolling R² of OLS weekly hedge ratios

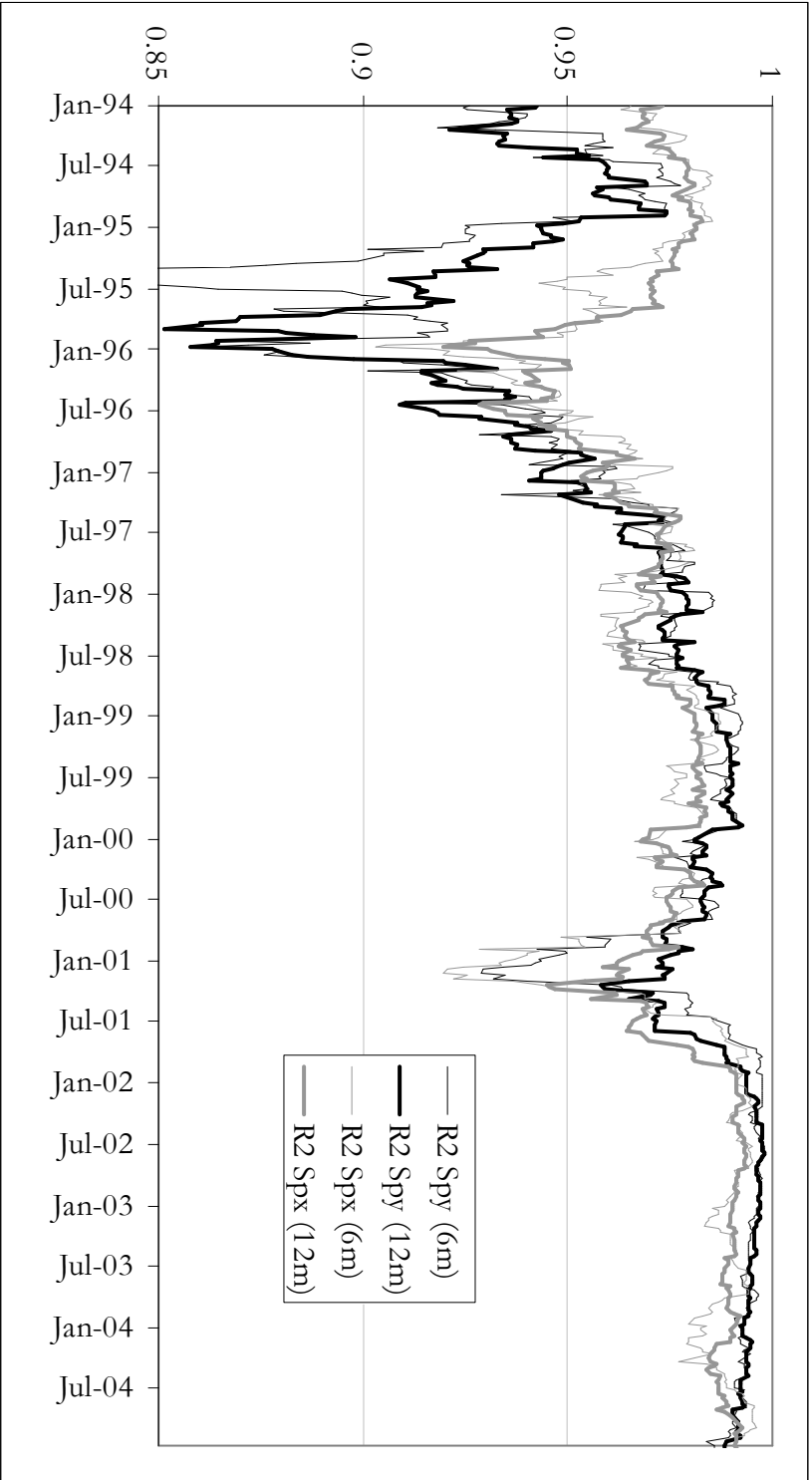


Figure 7: Volatility of the hedged portfolio

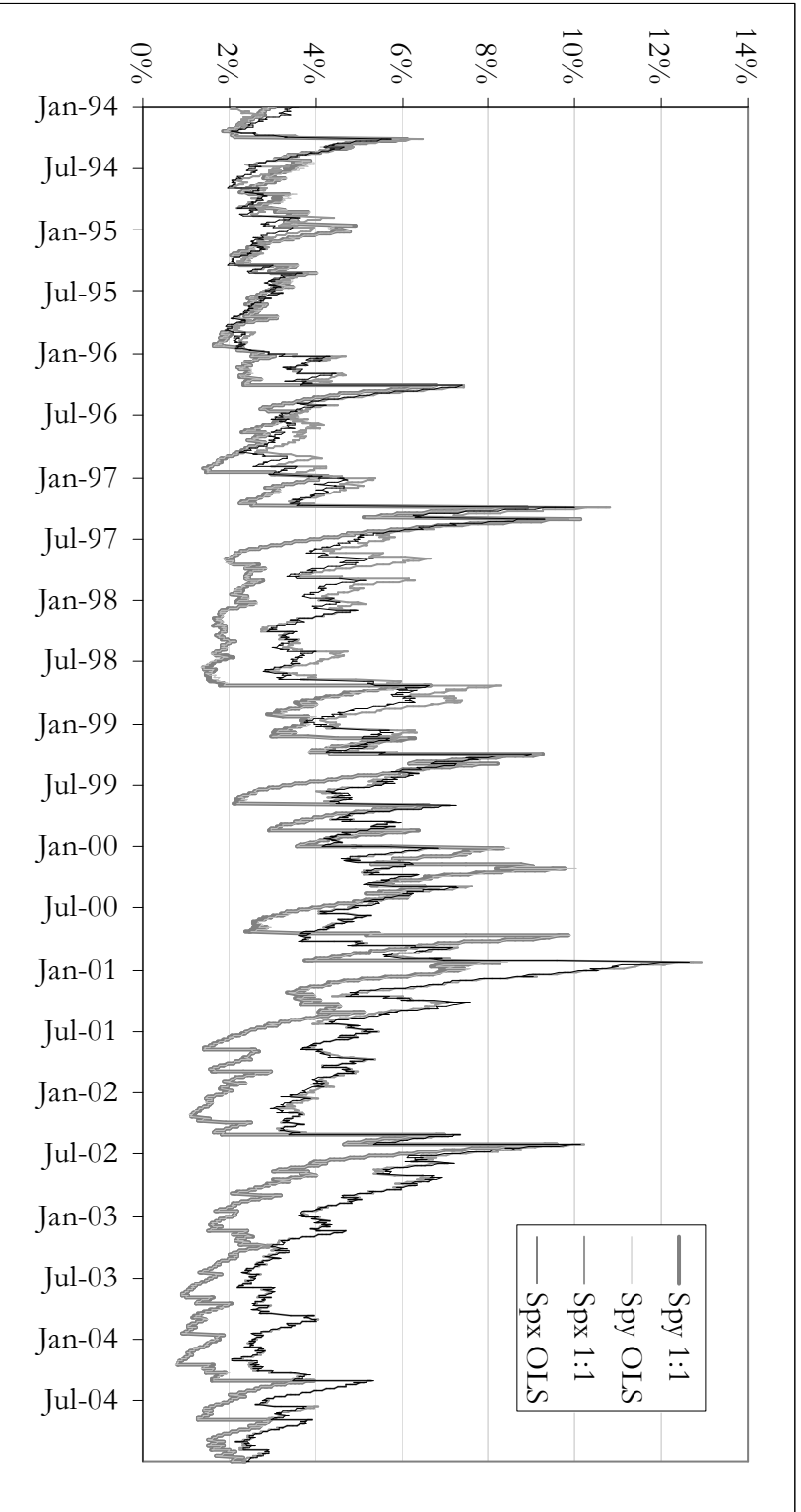


Figure 8: ECM residual based hedge ratio estimates (Spider)

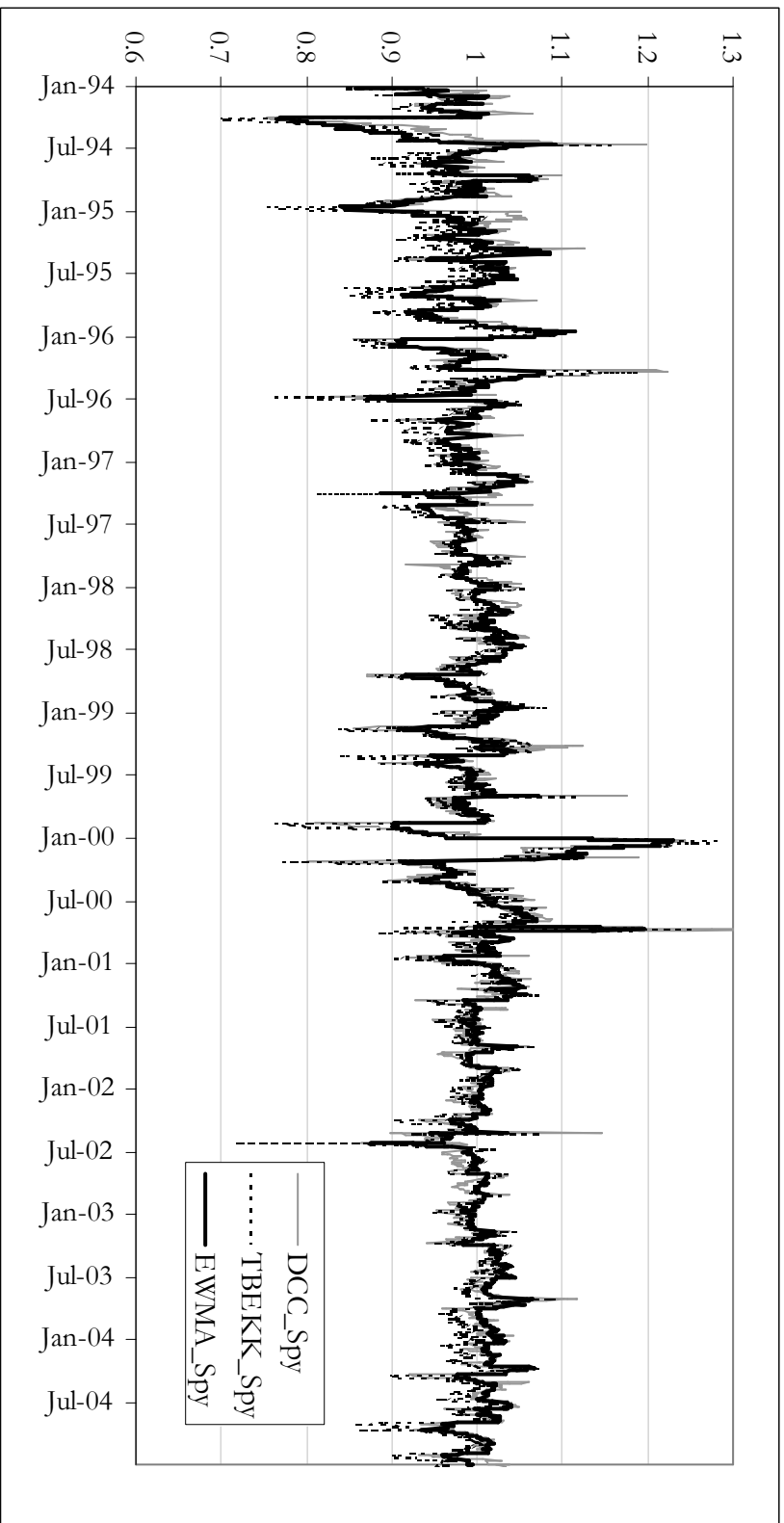


Figure 9: ECM residual based hedge ratio estimates (S&P500 index)

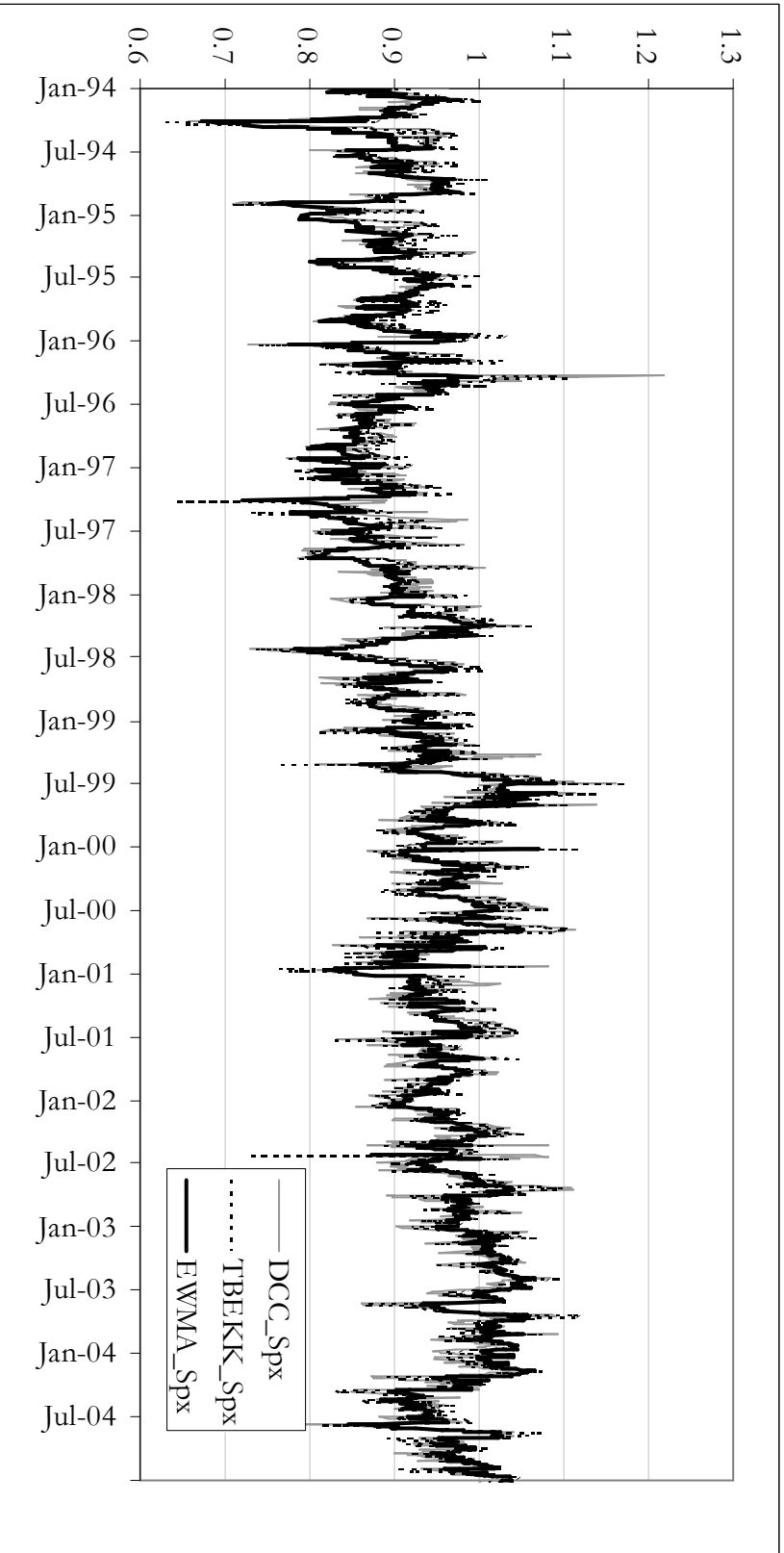


Figure 10: Effectiveness of daily hedges

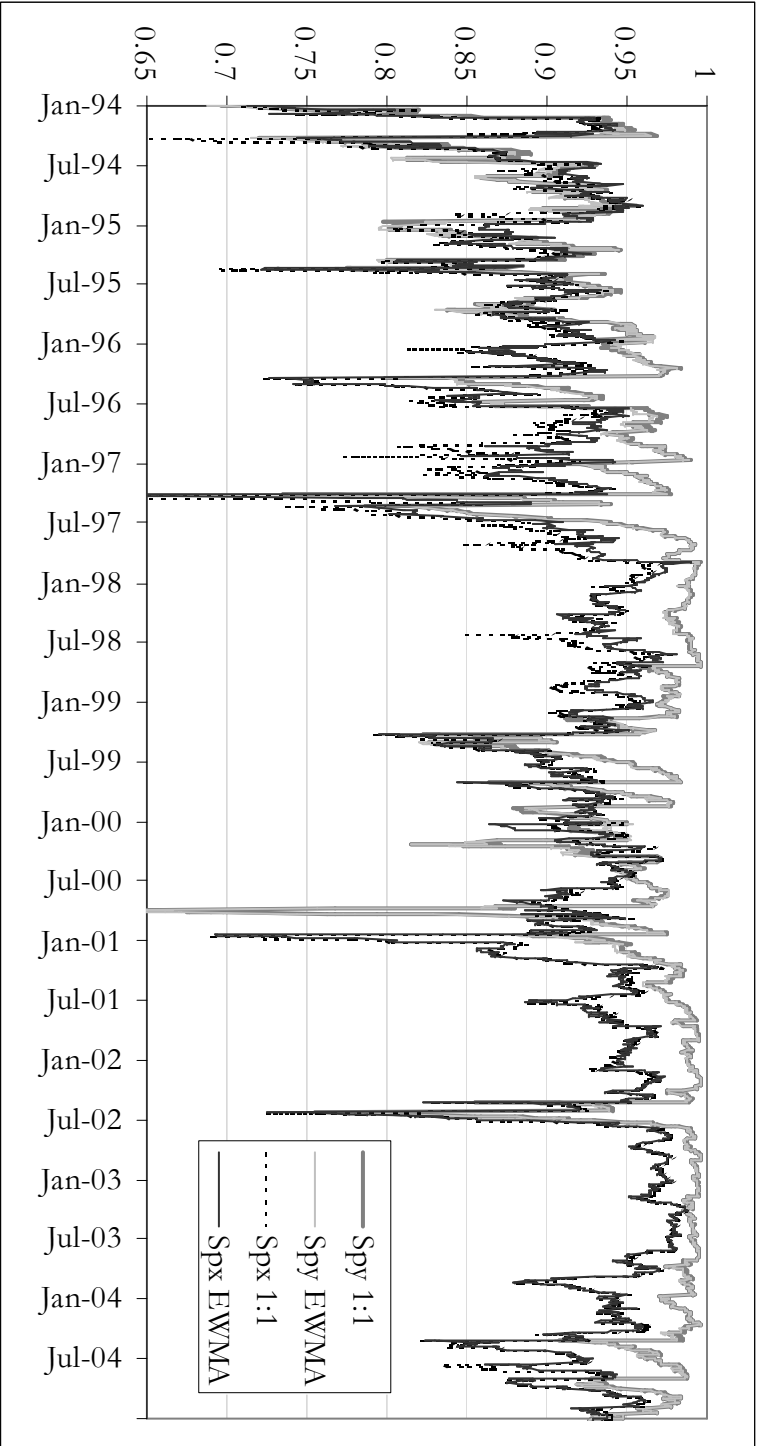


Figure 11: Effectiveness of weekly hedges for the S&P500 index

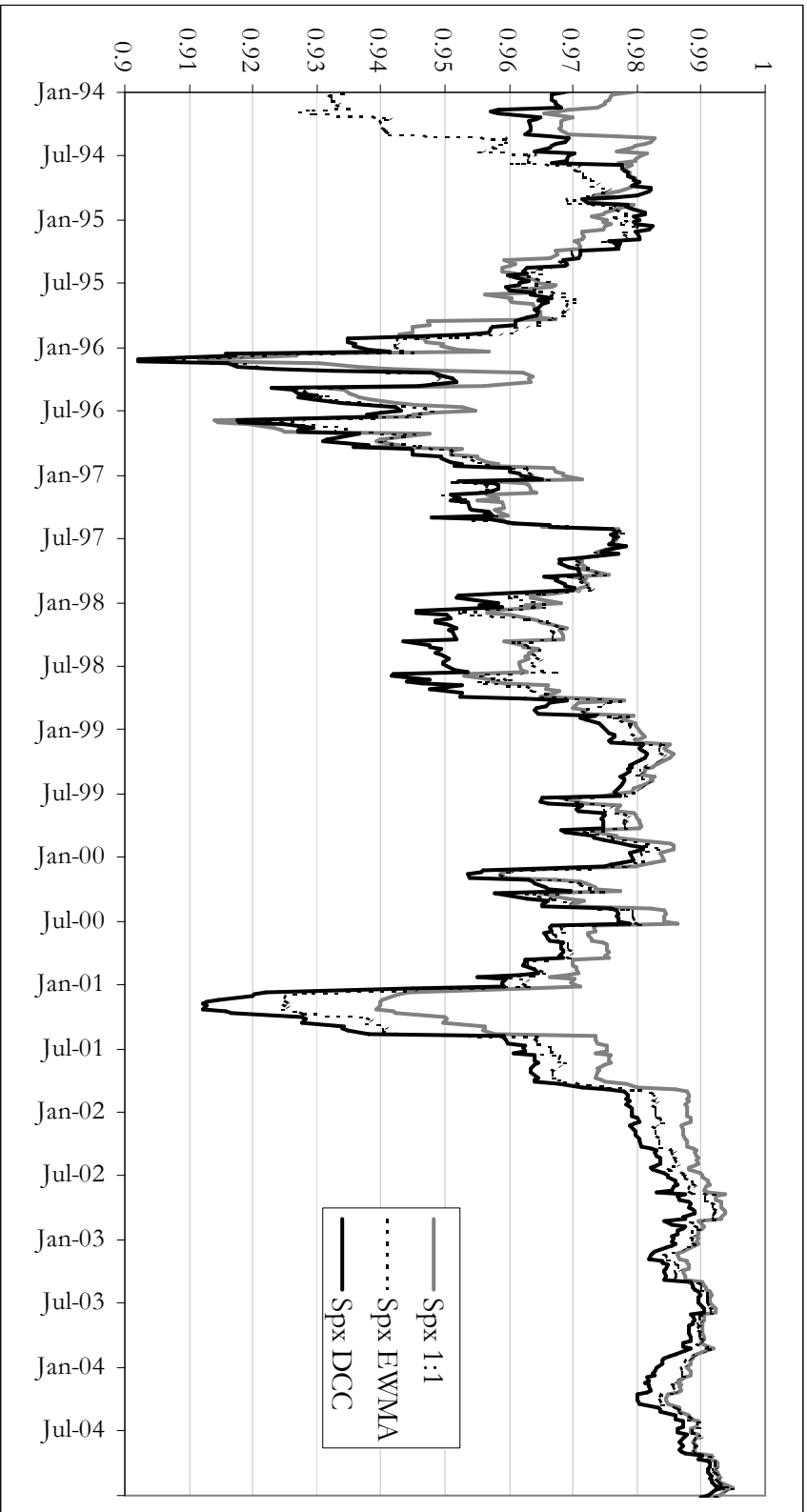


Figure 12: Daily DCC Conditional Correlation estimates

