Effectiveness of Minimum-Variance Hedging

The impact of electronic trading and exchange-traded funds.

Carol Alexander and Andreza Barbosa

The debate on econometric models for estimating the minimum-variance futures hedge ratio has run for many years. Hedging commodities is an interesting econometric problem because carrying costs are difficult to predict, and the basis can be high and variable, but stock indexes generally have much lower basis risk. Nevertheless econometric models for minimum-variance (MV) hedging of stock indexes continue to be the focus of a huge amount of empirical research.

We show that minimum variance hedging only provides an out-of-sample hedging performance that is superior to that of the naïve futures hedge in less developed markets, which have no active trading of ETFs or advanced electronic communications networks. Moreover, even when minimum variance hedging does out-perform a naïve hedge we find no evidence that complex econometric models, including time varying conditional covariances and error correction, can improve on the simple ordinary least squares hedge ratio.

A recent strand of the literature on market microstructure that relates to the impact of electronic trading on bid-ask spreads. Electronic trading reduces both human errors and, as smaller lot sizes become economically feasible, market impact. Also, given the discipline required to commit trading rules to execution algorithms, it increases objectivity. Moreover, electronic trading increases liquidity because transaction costs are reduced.

A recurrent proxy for transaction costs is the bid-ask spread, and several writers have tested the impact of electronic trading on reducing this spread. Early studies in New Zealand (Blennerhasset and Bowman [1998]) and Germany (Frino, McInish, and Toner [1998]), and on the cross-listing of Bund contracts in Germany and England (Pirrong [1996]), report lower bid-ask spreads in...

More recently, Chordia, Roll, and Subrahmanyam [2001] analyze the characteristics of U.S. equity market spreads, depths, and trading activities. They observe a downward trend in spreads and the opposite trend in depth and volume. They also find that bid-ask spreads respond asymmetrically to market movements, widening significantly in downward markets but narrowing only marginally in up markets, and they also argue that excessive market volatility can reduce rather than widen the bid-ask spread.

Copeland, Lam, and Jones [2004] conclude that screen trading does not improve the efficiency of FTSE, CAC, DAX, and KOSPI futures trading. Yet Fung et al. [2005] argue that electronic trading on the Hang Seng Composite Index attracts more informed traders to the futures market and increases information flow, in addition to reducing the bid-ask spread.


Thus conflicting claims on the advantages of electronic systems for reducing transaction costs and providing effective liquidity, especially during highly volatile periods, are frequent in academic research. Yet a recent survey by Burghart [2006] on global trends in trading volumes and average bid-ask spreads provides a convincing argument that the growth of electronic trading systems since 1999 has been the driving force behind dramatically increased trading volumes and huge reductions in bid-ask spreads in the last six years. He shows that while average volatility-adjusted bid-ask spreads on several electronically traded futures have steadily declined to between 60% and 90% of their level in 1999, those on pit-traded contracts such as soybeans and crude oil have not dropped at all.

While the effect of electronic trading on market efficiency remains a point of academic debate, there is a consensus view that cash market efficiency is enhanced with introduction of an exchange-traded fund (ETF) or iShare on the index. Ackert and Tian [2000, 2001], Switzer, Varson, and Zghidi [2000], Chu and Hsieh [2002], and Kurov and Lasser [2002] argue that an ETF or iShare contract facilitates spot-futures arbitrage, thus enhancing cash market efficiency and reducing the no-arbitrage range for the future about its fair value.

In the presence of high trading costs and costly information, we might see temporary divergence from the equilibrium relation between spot and futures prices. But, as trading costs drop and spot-futures arbitrage is facilitated by an ETF, the correlation between spot and futures returns increases, and basis risk declines.

Hence the development of both index ETFs and an advanced electronic trading networks may reduce the efficiency of a minimum-variance hedge ratio compared to a naive hedge, which matches the short position in the future exactly to the spot position. We shall show that no significant gains can now be made from MV futures hedging of some major stock indexes, although MV hedging may still improve on a one-to-one hedge on less efficient exchanges.

Furthermore, we show that on those exchanges where MV hedging may still be more effective than a one-to-one hedge, it is not possible to distinguish which econometric model most efficiently reduces the variance. Finally, our results also support the growing evidence (e.g., from Copeland and Zhu [2006]) that more sophisticated econometric models such as GARCH introduce too much noise to provide cost-effective hedges.

The theoretical and empirical methodology in this article extends previous research in two significant ways:

- Lien [2005] proves that it is not appropriate to evaluate the hedging performance of conditional variance minimization using the unconditional effectiveness measure of Ederington [1979], even though it is commonly used in most research on this subject. Since performance is sample-specific (it depends on both the estimation and the evaluation samples), we introduce a conditional measure of hedging effectiveness;
- We use this conditional effectiveness measure to examine the evolution of hedging efficiency over many years, before and after the development of electronic trading platforms. Our results are based on an out-of-sample performance test on nearly 4,000 observations of most of the seven indexes, considerably larger than in any other published research.

LITERATURE ON MINIMUM-VARIANCE HEDGING

Johnson [1960] was the first to derive the number of futures contracts necessary to hedge a certain spot posi-
tion based on minimizing the variance of the hedged portfolio. Much of the debate that followed concerned whether the minimum–variance (MV) criterion is appropriate, because it is based on a quadratic utility function, which is only one of many possible objective functions. Other utility functions (as in Cecchetti, Cumby, and Figlewski [1988]) or alternative hedging objectives may be applied. For instance: Howard and D’Antonio [1984] design a hedge to maximize the Sharpe ratio; Cheung, Kwan, and Yip [1990], Lien and Luo [1993], and Lien and Shaffer [1999] minimize the mean–Gini coefficient; and Eftekhar [1998], Lien and Tse [1998, 2000], and Mattos, Garcia, and Nelson [2006] employ objectives that include minimization of the generalized semivariance or higher lower partial moments.

Lien and Tse [2002] and Chen, Lee, and Shrestha [2003] review the huge literature on futures hedging. Many researchers use MV hedge ratio estimation based on an advanced econometric model with time–varying hedge ratio given by the ratio of the conditional covariance of spot and futures returns to the conditional variance of the futures returns. Seminal work by Baillie and Myers [1991] concludes that the generalized autoregressive conditional heteroscedastic (GARCH) model performs better than other dynamic or constant hedges, given the time–varying nature of the conditional distributions of commodities returns and their futures contracts.

Moschini and Myers [2002] reject the hypothesis of a constant hedge for the corn weekly series, under the assumption that spot and futures prices have GARCH effects. They also reject the hypothesis that seasonality and time to maturity account for all the variation in the optimal hedge ratios.

Chan and Young [2006] incorporate a jump component in the bivariate GARCH model to hedge the copper market, and find that the jump GARCH hedge performs better than the constant hedge for both daily and weekly frequencies. Other more advanced models, such as the Markov switching GARCH of Lee and Yoder [2005], also appear useful for hedging commodity prices.

It is not only in commodity markets that advanced econometric models may produce more efficient MV hedge ratios. Bhattacharya, Sekhar, and Fabozzi [2006] show that the cointegration GARCH model provides a powerful means of pricing and hedging mortgage–backed securities (MBS); it is more efficient than standard regression because it captures the dynamics between MBS and Treasury note futures in a low interest rate environment. Their hedging results show that the cointegration GARCH model is substantially better than the regression–based model at hedging various MBS with a ten–year Treasury note futures contract and another MBS, with the cross–hedge effectively accounting for negative convexity. This extends work by Koutmos and Pericli [1999], who test the effectiveness of the cointegration GARCH hedge ratios without a cross–hedge.

In fixed–income markets, the underlying and hedging contracts often differ, so a portfolio of hedging instruments may be used. The underlying may be less liquid than the hedging instruments; hence their correlation will rarely be close to unity. And in commodity markets, the basis may be extremely volatile, and prices may not follow a random walk. In these circumstances, advanced econometric models can be useful for computing the most efficient hedge ratio.

Many econometricians, however, apply similar MV hedge ratios to hedging stock indexes—and it is the usefulness of this research that we question in this article. The case for MV hedging of stock indexes is much less sound than it is for commodities, fixed–income securities, and indeed any asset where it is likely that the correlation between the underlying and the hedging instrument is high, but far from perfect. Stock trading is now highly efficient on many exchanges, and the basis risk on stock indexes is usually very low indeed.

Seminal research on minimum–variance hedging of stock indexes is by Figlewski [1984], who analyzes futures cross–hedging and hedging with the S&P 500 index between June 1982 and September 1984. Subsequent authors investigate the effect of dividend yield (Graham and Jennings [1987]), futures mispricing (Merrick [1988]), duration and expiration effects (Lindahl [1992]), and investment horizon (Geppert [1995]). Sutcliffe [2005] provides a comprehensive review of the literature on hedging stock indexes with futures.

Hedgers of stock indexes include stock market makers, equity hedge funds, and indeed any investor aiming to neutralize the market risk factor derived from a mandatory exposure in a portfolio. But by hedging an exposure to a stock index the investor is also giving up potential returns; in other words, the hedge is not costless. A direct cost of rebalancing the hedged portfolio arises when MV hedge ratios are employed. While the direct cost per transaction is likely to be low, the cumulative cost of MV–hedging large positions over a long period may be significant.

Advanced econometric models can produce hedge ratios that vary excessively over time, as shown by Lien,
models with traditional GARCH, ECM, and OLS methods. Nomikos [2004] compare Markov switching GARCH computing minimum-variance hedge ratios. Alizadeh and Nomikos [2004], Miffre [2004], and Yang and Allen [2005]. Thus increased transaction costs could offset any potential gain in efficiency. So, even if basis risk is high enough to warrant the use of MV hedge ratios, the costs may well be greater than the benefits.

Index spot and futures prices typically have a unit root, and error correction hedging models will take account of the basis convergence. Garbade and Silber [1983], Myers and Thompson [1989], and Ghosh [1993] take cointegration and the lead-lag relationship between cash and futures prices into account. Kroner and Sultan [1993] and Miffre [2004] incorporate conditionality in the available information with error correction models, but Lien [2004] has proven that omission of the cointegration relationship should have minimal impact on hedging effectiveness.

Other research aims to demonstrate the superiority of sophisticated dynamic hedge ratios for hedging stock indexes with futures. Using daily prices from June 1988 through December 1991, Park and Switzer [1995] show that a symmetric bivariate GARCH hedge ratio outperforms the constant hedge ratio for the S&P 500, MMI, and Toronto 35 stock indexes. Tong [1996] and Brooks, Henry, and Persand [2002] support this general result, but the latter, using daily prices on the FTSE 100 index and futures contracts from January 1985 through April 1999, find no improvement in hedging efficiency when asymmetric volatility responses are added to the GARCH model.

Choudhry [2003] compares naive, ordinary least squares, and GARCH hedge ratios for stock indexes in Australia, Germany, Hong Kong, Japan, South Africa, and the United Kingdom, with a two-year out-of-sample period between January 1998 and December 1999 to conclude that GARCH models perform the best. Floros and Vougas [2004] also find that GARCH hedge ratios perform better than OLS and vector error correction models (ECM) for hedging the Greek stock index market between 1999 and 2001. However, Laws and Thompson [2005], although apply OLS, GARCH, and exponentially weighted moving average (EWMA) hedge ratios to index tracking portfolios from January 1995 through December 2001, and conclude that the EWMA method performs the best.

More recent research investigates hedging efficiency using even more advanced econometric techniques for computing minimum-variance hedge ratios. Alizadeh and Nomikos [2004] compare Markov switching GARCH models with traditional GARCH, ECM, and OLS methods using weekly prices on the S&P 500 and FTSE 100 markets from 1984 through 2001, using a one-year out-of-sample period. They conclude that the Markov switching GARCH outperforms all other models in the FTSE market, and both GARCH models are better for the S&P 500.

Dark [2004] examines the bivariate error correction GARCH and fractionally integrated GARCH models applied to the Australian All Ordinaries Index, finding that these produce ratios that are better than the OLS and naive hedge ratios, a result that is supported by Yang and Allen [2005]. Yet there is no evidence that fractional integration improves the effectiveness of the GARCH model. Their out-of-sample period runs over three months only, ending in October 1999.

Finally, Lai, Chen, and Gerlach [2006] develop a copula threshold GARCH model to estimate optimal hedge ratios in the Hong Kong, Japan, Korea, Singapore, and Taiwan indexes. They report their model improves on traditional OLS in three of the five markets. Copeland and Zhu [2006] compare various dynamic ratios with the standard OLS hedge ratios for six equity markets (Australia, Germany, Japan, Korea, the U.K., and the U.S.). They argue that there are no clear benefits of using more sophisticated hedging models. Our study will add further weight to this argument.

An important critique of all this research is presented by Lien [2005]. The recurrent measure to evaluate hedging performance is the unconditional effectiveness measure proposed by Ederington [1979], but Lien proves that this measure is inappropriate when spot and futures prices are cointegrated. We extend Ederington’s methodology by computing a conditional effectiveness measure that allows one to evaluate the dynamic characteristics of the effectiveness of different hedging strategies.

SURVEY OF TRADING CHARACTERISTICS IN INDEX MARKETS

We investigate futures hedging effectiveness in seven stock indexes that have different trading characteristics: the Nasdaq 100 and S&P 500 indexes in North America, the FTSE 100 and CAC 40 indexes in Europe, the Hang Seng Composite and Kospi 200 indexes in Asia, and the Ibovespa index in South America. Nasdaq is arguably one of the most efficient and advanced electronic exchanges, and we compare it with the Hang Seng Composite, the Ibovespa, and the Kospi 200 where the electronic platforms are at an earlier stage of development. We also include the S&P 500 and the FTSE 100 because these have been the focus of much previous academic research.
U.S. Exchanges

Even though transaction costs on the technology stocks in the Nasdaq 100 index are relatively high, the Nasdaq exchange has a very highly evolved electronic communications network (ECN). The Nasdaq was originally a network of dealers, but brokers introduced an ECN during 1996–1997, and by 2002 even super-montage consolidated quotes had been introduced. In this sense the Nasdaq is more efficient than either the London or the New York stock exchanges.

Total trading volume on the Nasdaq 100 stocks during 2005 was very high (432,504 million USD), and since April 1999 there has also been a very liquid exchange-traded fund (the Cubes) on the Nasdaq 100 index. In terms of assets under management, the Cubes is the second-largest ETF in the U.S., but trading volume during 2005 actually exceeded that on the Spider, averaging over 90 million USD per day.

The S&P 500 stocks are traded on both the New York and Nasdaq exchanges; total trading volume amounted to 483,815 million USD during 2005. This index also has one of the most actively traded index ETFs in the world: the so-called Spider (i.e., Standard & Poor's depositary receipt). The American Stock Exchange released the Spider in 1993, and it also started trading on the New York Stock Exchange (NYSE) in 2001. During 2005 the average daily trading volume on the Spider was over 60 million USD, and by December 2005 the fund had a colossal 59 billion USD under management. This represents nearly one-quarter of the entire U.S. market in passive ETFs.

Another factor contributing to the efficiency of both the NYSE and the Nasdaq is that in April 1999 the U.S. Securities and Exchange Commission introduced new regulations governing trading and execution on electronic order books, requiring market makers to compete fairly with limit orders.

European Exchanges

FTSE 100 stocks trade on the London Stock Exchange (LSE). Some trading is still conducted by dealers even though the LSE electronic communications network, which is called the stock exchange trading service (SETS), was introduced in October 1997. The move toward electronic order book trading was slow initially, because the LSE lacked confidence in the system, but by the year 2000 a large proportion of trading was over SETS.

During 2005 total trading volume on FTSE stocks was 395,070 million USD on bid-ask spreads of around 25 basis points, depending on the share. Efficiency on the FTSE market is further enhanced by trading on the iFTSE 100 index share, although this contract is not nearly as liquid as many of the U.S. index ETFs.

The CAC 40 futures contract has its spot trading on Euronext, a totally integrated European cross-border market that now encompasses the LIFFE and Paris, Belfox, Amsterdam, and Lisbon exchanges. Since June 1, 1998, Euronext has operated only electronic trading, and the CAC 40 future is one of the most actively traded contracts on this exchange. An ETF for the CAC 40 index (the Lyxor ETF CAC 40) was launched in January 2001, and by the end of 2005 it had over 3 billion euros of assets under management.

Asian and South American Exchanges

For examples of markets where electronic trading is less developed than the U.S. and European exchanges, and index ETFs are not actively traded, we consider the Hang Seng Composite Index (HSCI) in Hong Kong, the Kospi 200 in Korea, and the Ibovespa index (IBOV) index in Brazil.

The HSCI stocks are listed on the Hong Kong Stock Exchange where brokers have operated an electronic auto-matching system since 1993. Futures trading migrated from the pit to an electronic platform in June 2000. The Hong Kong auto-matching system has recently been enhanced, to upgrade the limit order system, but it still has fewer advanced features than SETS and is less regulated than the ECNs in the U.S. Total trading volume on Hang Seng stocks during 2005 was only about 232,808 million USD, which is about 60% of the trading volume on the FTSE 100 and an even smaller fraction of the trading volume on either the S&P 500 or the Nasdaq 100.

An ETF on HSCI, the Tracker Fund of Hong Kong, was launched in November 1999. Even by 2005, the daily average trading volume on this fund was a mere 2.66 million USD, a small fraction of the volume traded on the Cubes or the Spider.

The Kospi 200 futures contracts began trading at the Korean Exchange on May 3, 1996. The trading of the contract reached a volume of nearly 34 million contracts in 2005 with a trading value of nearly 18 billion USD. An ETF on the Kospi 200 index (the Kodex 200) started trading in October 2002. Although it was introduced much later than the ETF on the HSCI, there is a much higher trading volume on the Kodex 200, and by the end of 2005 it had 800 million USD in assets under management.

The Korean Exchange also trades all contracts through an electronic system. According to the 2005
Annual Report of the World Federation of Exchanges, it was the fifth-largest exchange for trading of index futures contracts in 2005, after CME, Eurex, Euronext, and the National Stock Exchange of India.

The Ibovespa stocks are listed on the Sao Paulo Stock Exchange, and the futures contract is traded at the BM&F Futures Exchange. The Brazilian futures Ibovespa contract is traded in a hybrid market with both open outcry and electronic systems. The number of contracts traded on Ibovespa reached 6,065,361 in 2005. The BM&F is still in the process of moving the futures contract to trade solely on an electronic platform but specific limited maturities are traded on the floor. The Ibovespa does not have a tracker fund with shares traded on the exchange. The only ETF in the Brazilian market is benchmarked to the IBX index, which is highly correlated with the Ibovespa.

DATA AND THEIR CHARACTERISTICS

We use daily closing prices on the S&P 500, FTSE 100, HSCI, and their corresponding futures contracts from April 19, 1994. The Ibovespa dataset starts August 2, 1995, the Kospi 200 data on May 6, 1996, the Nasdaq 100 on April 15, 1996, and the CAC 40 data on January, 8 1999. The last trading day is April 19, 2006, in all datasets.

We divide each sample into two periods, before and after April 1, 1999. We choose this date because it marks the introduction of SEC regulation on ECNs. If U.S. and European market trading was less efficient in the pre-1999 subsample, we may find that minimum-variance hedging was more efficient then, compared with the post 1999 subsample.

Exhibit 1 summarizes the descriptive statistics of the two subsamples. For the first subsample we observe high average returns as a result of the bull market of the 1990s. As the technology bubble burst in the second subsample period, average returns were considerably lower, with negative values for the Nasdaq 100 and FTSE 100. The highest volatility is observed in Ibovespa during the first subsample, as this period includes the period immediately after the Brazilian stabilization plan. Volatility of the HSCI and Kospi 200 was also extremely high in the first subsample due to the Asian crisis in 1997.

All spot market returns are highly correlated with the corresponding futures returns. As expected, this correlation is the highest on the Nasdaq 100 and CAC 40 and the lowest on the HSCI and Kospi 200. For all the

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* Nasdaq sample I starts April 15, 1996. Ibovespa sample I starts August 2, 1995. Kospi 200 sample I starts May 6, 1996. The CAC sample I data are omitted since they start only January 8, 1999. The Johansen maximum eigenvalue cointegration test has a 1% critical value of 6.65. When spot and future prices are cointegrated, error correction models for MV hedge ratios should apply.
indexes except the Nasdaq 100 and Ibovespa, spot-futures returns correlation is notably higher during the second period. Finally, since all the spot and futures prices are cointegrated according to the Johansen maximum eigenvalue test (at a 1% critical value of 6.65), error correction models for MV hedge ratios should apply.

We should note there is a discrepancy between closing times in most cash and futures markets. For instance, in the S&P 500, Nasdaq 100, and Kospi 200, the cash market closes 15 minutes before the futures market. In fact, only the Ibovespa regular trading sessions of the futures and cash markets have synchronized closing times. Hence the use of daily closing prices on the spot index and the index future is likely to produce a downward bias on returns correlation and an upward bias on basis risk. Clearly this could result in the conclusion that MV hedge ratios are more effective than they really are.

The lack of synchronous daily data must have affected results in many other empirical studies that use daily closing prices to demonstrate the superiority of econometric models for estimating MV hedge ratios. In our study, however, we aim to support the hypothesis that MV hedge ratios cannot improve on the naive hedge once electronic trading has been fully developed and an ETF has become established. Hence non-synchronous data are not so much of an issue because they will tend to bias results in favor of the alternative hypothesis.

ECONOMETRIC MODELS FOR SHORT-TERM FUTURES HEDGING

Two questions arise with respect to futures hedging. The first is how to estimate the optimal number of futures contracts, and the second is how to measure the efficiency of the hedging strategy. These two questions are integrally related and should be tackled together.

The minimum-variance hedge ratio is defined as the number of futures per unit of the spot asset that will minimize the variance of the hedged portfolio returns. We calculate four different MV hedge ratios as follows:

1. OLS: The estimated slope coefficient in the simple OLS regression:
   \[ s_t = \alpha + \beta f_t + \epsilon_t \]  
   where \( s_t \) and \( f_t \) denote the daily log returns on the spot index and the index future.

2. ECM: A lagged disequilibrium term and lagged dependent variables are added in a bivariate vector error correction mechanism. The equation for the spot returns is:
   \[ s_t = \alpha + \beta f_t + \beta s_{t-1} + \beta f_{t-1} + \lambda z_{t-1} + \epsilon_t \]  
   and \( z \) is the difference between the log of the futures price and the log of the stock price. Here the OLS estimate \( \hat{\beta} \) is the minimum-variance hedge ratio.

3. EWMA: Similar to the ordinary least squares ratio, but uses exponentially weighted average estimates of the futures returns variance (\( \hat{\sigma}_f^2 \)) and of the spot and futures returns covariance (\( \hat{\sigma}_{sf} \)). That is, we put:
   \[ \hat{\sigma}_{sf}^2 = \lambda \hat{\sigma}_{sf}^2 + (1-\lambda) s_{t-1} f_{t-1} \]  
   and these give a time-varying estimate of the hedge ratio as:
   \[ \hat{\beta}_i = \frac{\hat{\sigma}_{sf}^2}{\hat{\sigma}_f^2} \]  

4. GARCH: Calculated as in (3), but the variance and covariance estimates are obtained from a bivariate GARCH model. Here the two conditional mean equations are given by a bivariate vector error correction mechanism, following the general specification for cointegrated processes given by Engle and Granger [1987].

The models 1, 2, and 4 are estimated using the last six months of daily data, with one lag for the ECM. Model 3 is based on a smoothing constant of \( \hat{\lambda} = 0.95 \). These decisions carry a certain element of model risk, so we also examine results for different lags in (2) and different in-sample periods (three months, one year, and two years), and we also vary the value of \( \hat{\lambda} \) within reasonable limits. In 4, we estimate a variety of different bivariate GARCH models. While the numerical results vary, the general conclusions remain unchanged.

A traditional measure of hedging effectiveness, derived by Ederington [1979] and since applied in numerous empirical studies, is the percentage reduction in variance:
   \[ E = \frac{\sigma^2 - \sigma^2_w}{\sigma^2_w} \]  
   where \( \sigma^2 \) and \( \sigma^2_w \) denote the sample variance of the unhedged and hedged portfolio returns, respectively. Since the MV criterion is applied in-sample and the hedging effectiveness is tested out-of-sample, there is no guarantee
that MV hedging will produce more effective hedges, in terms of variance reduction, than the naive hedge.

Lien [2005] has emphasized the inadequacy of the regression $R^2$ to evaluate minimum-variance hedge ratios other than OLS. He has also proved that (4) will favor the OLS hedge ratio when spot and future prices are cointegrated. For this reason, and also to provide a time-varying measure of out-of-sample hedging effectiveness, we propose a conditional effectiveness measure:

$$E_t = \frac{\sigma_u^2 - \sigma_h^2}{\sigma_u^2}$$

(5)

where $\sigma_u^2$ and $\sigma_h^2$ denote conditional variances of the unhedged and hedged portfolio out-of-sample returns, respectively. For simplicity, the EWMA variance with a smoothing constant of 0.95 is used in (5).

**EMPIRICAL RESULTS**

For each hedging model we perform an out-of-sample analysis of hedging performance with daily rebalancing. Each day we estimate the MV hedge ratio to determine the futures position to be taken at the end of that day until the following day. The sample is then rolled one day, the hedge ratios reestimated, and the hedge rebalanced and held until the end of the next day. Each model is estimated using a six-month in-sample period starting April 19, 1994 (or April 15, 1996, for the Nasdaq 100, January 8, 1999, for the CAC 40, August 2, 1995, for Ibovespa, and May 3, 1996, for the Kospi 200), and rolling the estimation period forward one day at a time until we reach the end of the entire sample.

Exhibit 2 shows how different hedge ratios have evolved over the sample period for the FTSE 100. For clarity the hedge ratios for the other indexes are not shown, but in each case the ECM hedge ratio displays similar time-varying characteristics as the OLS ratio except that it is generally closer to one, and the EWMA and GARCH ratios are much more variable because they are based on conditional covariances. Hence both EWMA and GARCH models would require considerable rebalancing at the daily frequency, a feature also observed by Lien, Tse, and Tsui [2002], Choudhry [2003], Poomimars, Cadle, and Theobald [2003], Alizadeh and Nomikos [2004], Miffre [2004], and Yang and Allen [2005].
In both the FTSE 100 and the S&P 500 the hedge ratios increase toward 1.0 over time. In the CAC 40 and Nasdaq 100, the hedge ratios are very close to 1.0 over the entire period. As expected, the HSCI has the lowest average hedge ratio, and for the Ibovespa we cannot distinguish any trend. The Kospi 200 has lower hedge ratios in the first sample period, and they are particularly low during the Asian crisis; thereafter they increase toward unity.

Exhibits 3–5 illustrate the differences between the conditional effectiveness measures given by the four different MV hedge ratios and the naive hedge, a positive value, thus indicating that the MV ratio performs better than the one-to-one ratio. The time series in Exhibit 3 for the FTSE 100 index shows very clearly that, when effectiveness is measured in a time-varying framework, no significant variance reduction from MV hedging beyond the variance reduction offered by the naive hedge has been possible since 2000. Before then, the MV hedge ratios offered greater variance reduction than the naive hedge, although it is not possible to decide whether the ordinary OLS, the ECM, the EWMA, or the GARCH hedge was the better strategy.

Equivalent plots for the S&P 500, and the Kospi 200 indexes (not shown) indicate very similar characteristics. MV hedging appears more efficient than one-to-one hedging prior to 2000, but since then no significant variance reduction from MV hedging is apparent. It should also be noted that an extremely high volatility on the Kospi 200 index may have contributed to higher spreads and thus made MV hedges more efficient than the naive hedge during the Asian crisis in 1997.

Exhibits 4 and 5 are in stark contrast to each other. From Exhibit 5 we see that the Nasdaq exchange is so efficient that MV hedging has never been able to reduce variance significantly compared to the naive hedge, except for a few short and isolated periods in the sample. These are periods of high volatility on the Nasdaq 100 index, during the Asian crisis in 1997, the Russian crisis in 1998, and the burst of the technology bubble in 2000. At these times there is a small improvement in the conditional efficiency of the MV hedge ratios compared to the naive hedge. It is apparent, however, that trading on the Nasdaq is efficient enough to ensure that transaction costs and therefore basis risk remains low.

The equivalent exhibit for the CAC 40 index (not shown) also indicates zero improvement from MV hedging, although only the second subsample is available. For the Ibovespa (also not shown), we find a few isolated

**Exhibit 3**

difference in conditional efficiency—FTSE 100
**Exhibit 4**
Difference in Conditional Efficiency—Nasdaq 100

**Exhibit 5**
Difference in Conditional Efficiency—HSCI
periods when a MV hedge can be marginally more effective than the naive hedge, and these are also associated with excessively high volatility in the market. Exhibit 5 shows that for the HSCI all MV hedges can dramatically improve on the naive hedge, even during the latter part of the sample.

Exhibit 6 reports the volatilities and the Ederington measure for all the out-of-sample hedged portfolio returns in both subsamples. Except in the Nasdaq 100 and Ibovespa, where there is very little difference in the efficiency of different hedge ratios, the naive hedge is clearly less efficient than the MV hedges during the first period. Hedging in the Kospi 200 and the HSCI is less effective than for the other indexes, although it improves during the second subperiod, and for these indexes the naive hedge remains less effective than the other hedges even during the second subperiod. During the second period and in the other indexes there is no significant difference between the hedge portfolio returns distributions, whatever the hedge ratio used.

The results in Exhibit 6 are the probability values of Kolmogorov-Smirnoff (KS) tests for the hypothesis that the MV hedge portfolio returns are drawn from the same distribution as the naive hedged portfolio returns. During the first subperiod, the KS statistics for FTSE 100 and HSCI indexes are highly significant, and those for the Kospi 200 index also have low probability values.

The probability values of the KS statistics are uniformly greater in the second subperiod, indicating that the distributions of MV hedged portfolio returns are moving closer to the distribution of naive hedged portfolio returns. During the second sub-sample the probability values of the KS statistics for HSCI, Ibovespa, and Kospi 200 are much lower than they are for the U.S. and European indexes. Compare, for instance, the probability value of 0.9997 for the GARCH hedge of the S&P 500, with the probability value of 0.1263 for the equivalent hedge of the HSCI.

We also use KS statistics to test for any significant difference between hedged portfolio returns based on different MV methods. These results have been omitted, for brevity, but all this evidence is available from the authors by request. No significant results are found so we must conclude that there is no discernible difference between any of the MV hedging strategies for any of the indexes in either subperiod.
SUMMARY AND CONCLUSIONS

Our study argues against the use of MV hedge ratios for short-term futures hedging of stock indices in U.S. and European markets, where advanced electronic trading platforms have increased trading volume and reduced transactions costs, and actively traded index have further increased trading efficiency. We have shown that MV ratios in these markets have offered no discernable improvement on the naïve futures hedge since the turn of the century.

However in markets where trading is less efficient, such as the Hang Seng composite index, econometric models may still provide hedge ratios with more efficient variance reduction than the naïve hedge. Even so, we found no evidence to suggest that complex econometric models such as GARCH can improve on a simple OLS regression for estimating this hedge ratio.

This last finding accords with results in Poomimars, Cadle, and Theobald [2003], who compare the empirical performance of several dynamic and static models in seven markets (the S&P 500, Nikkei 225, FTSE 100, Japanese yen, British pound, gold, and silver). They conclude that hedging performance is similar for most models. Moosa [2003] also concludes that:

Although the theoretical arguments for why model specification does matter are elegant, the difference model specification makes for hedging performance seems to be negligible. What matters for the success or failure of a hedge is the correlation between the prices of the un-hedged position and the hedging instrument. Low correlation invariably produces ineffective hedges, whereas high correlation produces effective hedges irrespective of how the hedge ratio is measured [p.].

Our results are also consistent with previous empirical research that demonstrates that some MV models introduce too much noise to be effective for hedging purposes. The benefits of an active hedging strategy should be economically justifiable, yet these models do not account for transaction costs such as margins and commissions. When the costs of hedging are considered, the case against MV hedge ratios based on conditional covariances is strengthened even further.

We find that the more advanced the econometric model used, the greater the variability in the hedge ratio, and the more frequently the hedged portfolio would be rebalanced in practice. In this respect, we agree with Lence [1995], who argues that sophisticated econometric models for estimating MV hedge ratios provide negligible economic benefits and suggests that the effort dedicated to estimate better MV hedges “has been a waste of resources.”

ENDNOTE

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