Sources of Outperformance in Equity Markets

Mean reversion, common trends, and herding.

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Comparisons between the two main equity investment styles—active and passive—have a long history, much influenced by both academic research and the investment management industry. Active management seeks to outperform the market, usually through stock selection or market timing, while passive management is aimed at replicating the market's performance.

Replicating market performance through a passive strategy, most frequently in the form of indexing, relies on the principles of efficient markets and modern portfolio theory, where the only way investors can beat the market over the long run is by taking greater risk (see Fama [1970]). Active management has been shown to often underperform its passive alternative because of transaction costs and administrative fees, mostly in bull but also in bear markets (see Jensen [1968], Elton et al. [1993], and Carhart [1997]). For example, the S&P active/passive scorecard for year-end 2003 shows that the majority of active funds failed to beat their relevant index over the last five years.

The passive investment industry has witnessed remarkable growth in the last ten years, and a huge number of funds have pegged their holdings to broad market indexes such as the S&P 500.

Traditionally, indexing has targeted price-weighted and value-weighted indexes, which are easy to replicate with portfolios constituting the entire set of stocks, and mirroring the benchmark weights. Such portfolios are self-adjusting for changes in stock prices and do not require

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any rebalancing, provided there are no changes in the index composition or in the number of shares in each issue.

Despite the self-replication advantage, holding all the stocks in the benchmark may not always be desirable or possible, mainly because of difficulties in purchasing odd lots to exactly match the market weights, or the increased transaction costs and market impact related to trading less liquid stocks. More involved strategies are also required to track equally weighted indexes, since frequent rebalancing is required in order to maintain equal dollar amounts in each stock.

Larsen and Resnick [1998] provide a thorough empirical investigation of the relationship between the indexed portfolio's composition and tracking performance. Their results show that value-weighted indexes are easier to replicate than equally weighted indexes, and capitalization dominates other stratification criteria such as industry classification.

Given the disadvantages of direct replication, recent research has focused on developing optimization models for passive investments. Conventionally, tracking strategies using fewer stocks are constructed on basic capitalization or stratification considerations. Optimization techniques have also been developed using objective functions based on correlation of the portfolio returns with the benchmark, mean deviation of the tracking portfolio returns from the benchmark, the variance of this deviation (often referred to as tracking error), or transaction costs. Some examples are Rudd [1980], Meade and Salkin [1989], Adcock and Meade [1994], Connor and Leland [1995], Larsen and Resnick [1998, 2001], and Alexander [1999].

We contribute to this line of research by proposing a general portfolio construction model based on principal components analysis. The model identifies, of all possible combinations of stocks with unit norm weights, the portfolio that captures the greatest amount of the total joint variation of stock returns. Such a property makes it the optimal portfolio to capture the common trend in a system of stocks while filtering out a significant amount of noise.

There is widespread use of statistical techniques in finance to model asset returns, especially in the context of factor models. Starting with Feeney and Hester [1967] and Lessard [1973], and including more recently Schneeweiss and Mathes [1995] and Chan, Karceski, and Lakonishok [1998], principal components and factor analysis have been used to examine common movements in stock returns. These methods are seen as alternatives to fundamental approaches that relate the factors influencing financial asset returns to macroeconomic measures such as inflation, interest rates, and market indexes, or to company specifics such as size, book-to-market ratio, or dividend yield.

A great deal of statistical factor analysis has been devoted to tests of the arbitrage pricing model (Ross [1976]). In this case, historical returns are used to estimate orthogonal statistical factors and their relationship with the original variables. The construction of mimicking portfolios for the statistical factors has been formalized by Huberman, Kandel, and Stambaugh [1987]. Alternatives to standard principal components analysis have been developed, e.g., asymptotic PCA (Chamberlain and Rotschild [1983], Connor and Korajczyk [1986, 1988]) or independent component analysis (Common [1994]).

A common finding in the literature is that the first principal component of a group of stocks captures the *market factor* (Connor and Korajczyk [1988]; Chan, Karceski, and Lakonishok [1998]).[1] This assessment is based on two observations. First, provided that stock returns are reasonably correlated, they will have similar loadings on the first principal component, so a shock to this factor will generate a common trend in the system.

Second, the R^2 from a simple regression of an equally weighted portfolio of all stocks on their first principal component is usually found to be high, above 0.8, so that the first principal component explains to a great extent the returns on an equally weighted portfolio. This result can be extrapolated also to other types of indexes, such as price-weighted, provided that the returns of price-weighted and equally weighted indexes representing the same universe are generally highly correlated. Our portfolio construction model is based precisely on the resemblance of the first principal component of the stock returns to the market factor proxied by a traditional index.

The standard approach to constructing factor-mimicking portfolios uses the factor loadings in the stock selection process (e.g., Fama and French [1993]). Stocks are ranked according to their loading on a particular factor, and then a self-financed portfolio is set up with long positions on the stocks with the highest loadings on that factor and short positions on the stocks with the lowest loadings. Most frequently, there is no portfolio optimization, and equal dollar amounts are invested in each stock.

An alternative proposed by Fung and Hsieh [1997] for factor-mimicking portfolios considers, in the fund selection stage, only the funds that are highly correlated solely with the principal component for which the replica is constructed. After selecting the funds, portfolio weights are optimized so as to deliver the maximum correlation of the mimicking portfolio returns with the corresponding principal component.

In these two methods, principal components analysis is used as a selection technique, and portfolio construction is a separate stage, based either on a standard optimization, or on an arbitrary method such as equal weighting. We propose a different approach that constructs a portfolio replicating the first principal component directly from the normalized eigenvectors of the covariance matrix of stock returns. Such a portfolio, by construction, captures the greatest proportion of the variation in stock returns and filters out a significant amount of noise. It is naturally suited for a passive investment framework, requiring a fully invested portfolio of all stocks, but involving a very small amount of rebalancing trades because it captures only the major common trend in stock returns.

This procedure involves a single optimization, the one producing the principal components. Moreover, there is no arbitrary choice of the portfolio construction model, such as equal weighting of stocks.

To investigate portfolio performance, we use groups of stocks included in the Dow Jones Industrial Average (DJIA), the S&P 100, the FTSE 100, and the CAC 40 universes.

Portfolio performance is analyzed both before and after transaction costs. We examine return volatility and correlation and higher-order moments of return distributions, both from an overall perspective and conditional on market circumstances. Even if a benchmark does not enter the portfolio construction model, we follow convention and use both price-weighted and equally weighted indexes as benchmarks for portfolio performance.

Unsurprisingly, our results indicate that the first principal component captures the market factor, as it is highly correlated with the benchmark returns. Moreover, the factor weights prove to be very stable over time, so transaction costs are minimal.

What does come as a surprise is that, over long periods, the portfolio replicating the first principal component, while highly correlated with its benchmarks, has significantly outperformed both of them. We demonstrate that one cause of the outperformance is a mean reversion in returns for the group of stocks overweighted in the portfolio. We show that these are precisely the stocks that have been more volatile and have also been highly correlated as a group during the portfolio calibration period.

Subsequently, we observe two behavioral mechanisms that could explain the mean reversion for these stocks: the attention-capturing effect documented by Odean [1999] and the overreaction-based models of De Long et al. [1990b], Lakonishok, Shleifer, and Vishny [1994], and Shleifer and Vishny [1997]. Separately, our results show that the outperformance is related to a behavioral measure of investor herding toward the market factor, driving the mean reversion in stock returns.[2]

A decomposition of the strategy's outperformance into a market premium, a value premium, and a volatility premium reveals a time-varying structure. Throughout most of the period studied, the value component dominates the other two, but during the volatile periods of the last few years the strategy earned a significant volatility premium.

COMMON TREND REPLICATION MODEL

Principal components analysis (PCA), introduced by Hotelling [1933] in connection with the analysis of data in psychology, was recommended as an important tool in the multivariate analysis of economic data more than half a century ago (Tintner [1946]). The technique is now a standard procedure for an orthogonal transformation of variables, reducing dimensionality and the amount of noise in the data.

Given a set of k stationary random variables, X_1, X_2, ..., X_k, PCA determines linear combinations of the original variables, called principal components and denoted by P_1, P_2, ..., P_k, so that 1) they explain, successively, the maximum amount of variance possible, and 2) they are orthogonal. By convention, the first principal component is the linear combination of X_1, X_2, ..., X_k that explains the most variation. Each subsequent principal component accounts for as much as possible of the remaining variation and is uncorrelated with the previous principal components.

The i-th principal component, where $i = 1, ..., k$, may be written as:

$$P_i = w_{1i}X_1 + w_{2i}X_2 + ... + w_{ki}X_k \tag{1}$$

If we denote by $\mathbf{\Sigma}$ the covariance matrix of X, then:

$$\mathrm{var}(P_i) = \mathbf{w_i}'\,\Sigma\,\mathbf{w_i}$$
$$\mathrm{cov}(P_i, P_j) = \mathbf{w_i}'\,\Sigma\,\mathbf{w_j}$$

where $\mathbf{w_i} = [w_{1i}, w_{2i}, ..., w_{ki}]'$ and it is standard to impose the restriction of unit length for these vectors, i.e., $\mathbf{w_i}'\mathbf{w_i} = 1$.[3]

Note that these are, in fact, the eigenvectors of $\mathbf{\Sigma}$.

The spectral decomposition of the covariance matrix is $\mathbf{\Sigma} = \mathbf{W\Lambda W'}$, where $\mathbf{W}$ is an orthogonal matrix of eigenvectors (ordered according to the size of the corresponding eigenvalue), and $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues (ordered by convention so that $\lambda_1 > \lambda_2 > \ldots > \lambda_k > 0$). The principal components defined as $\mathbf{P} = \mathbf{XW}$ observe the conditions above.

Note that the variance of each principal component is equal to the corresponding eigenvalue, so the total variability of the system is the sum of all eigenvalues. To reproduce the total variation of a system of k variables, one needs exactly k principal components. When the first few principal components together account for a large part of the total variability, however, the dimensionality—and much of the noise in the original data—can be significantly reduced.

Since the principal components define a k-dimensional space in terms of orthogonal coordinates, the distances defined in the principal components space depend on the amount of correlation in the original variables. The higher the correlation in the original system, the better a principal component can account for the original joint variation and the wider the interpoint distances will be in that dimension.

The elements of the first eigenvector are the factor loadings on the first principal component in the variables representation given by the principal components. In a highly correlated system, these elements will be of similar size and sign. Consequently, when portfolio weights are directly proportional to the elements of the first eigenvector, as in Equation (2), the more correlated the stocks, and the more evenly balanced the portfolio.

In large stock universes, the first principal component captures the market factor, explaining to a great degree the returns of an equally weighted portfolio of all stocks. Motivated by these results, we propose a portfolio construction model based on replicating the first principal component of a set of stock returns.[4]

For a portfolio of k stocks, the portfolio weight of stock i is defined as:

$$w_i = w_{i1} \Big/ \sum_{j=1}^{k} w_{j1} \qquad (2)$$

where w_{i1} is the i-th element from the first column in the eigenvectors matrix ordered as above.

In the PCA framework, the first eigenvector is obtained, independently of the others, by maximizing the variance of the corresponding linear combination of stocks, under the constraint of a unit norm. Therefore, the portfolio based on the stock weights determined as in (2) is, of all possible combinations of k stocks with unit norm, the portfolio that accounts for the greatest part of the total joint variation of the k stocks. This property ensures that it is the optimal portfolio for capturing the common trend in a system of stocks. Considering that the model maximizes the variance of the portfolio under some constraint, it will overweight, relative to benchmark, the stocks that were both highly correlated and had higher than average volatility over the estimation period.

The common trend replication model is different from the traditional approaches to portfolio optimization (Markowitz [1952], Chan, Karceski, and Lakonishok [1999]; Jagannathan and Ma [2002]) in more than one respect. First, it maximizes and not minimizes portfolio variance, and this may appear counter-intuitive at first glance. When we combine this with a unit norm constraint on the factor loadings, however, the result is a balanced portfolio with a stable structure, which also explains most of the joint variance in the system of stocks.[5]

Second, it does not aim at stock selection, but rather at diversifying over the entire universe of stocks. All stocks will be represented in the portfolio replicating the common trend, and the portfolio will be fairly evenly balanced if there is a high level of correlation in the stock returns.

Finally, although this is a passive investment model, the benchmark does not enter into the methodology anywhere. This eliminates the problems associated with using an inappropriate benchmark in portfolio construction, but limits the relevance of traditional indexing performance measures such as tracking error, so caution is needed in interpreting such results.

DATA AND BENCHMARKS

To examine the properties of the portfolio replicating the first principal component, we use several data sets of daily closing prices of stocks included in the DJIA, the S&P 100, the FTSE 100, and the CAC 40 indexes as of the end of 2002. The longest data set comprises the 25 stocks currently included in the DJIA that have a history available for January 1980–December 2002. The length of the other data samples ranges from 1,600 daily observations for the S&P 100 (April 1996–June 2002) to 2,100 daily observations for the FTSE 100 (July 1994–December 2002).

The stock selection methodology may raise a concern of performance biases such as survivorship and look-ahead, because we select stocks with a long history in the index. For example, in the DJIA universe, four of the five stocks currently in the index that do not have a history going back to January 1980 are technology stocks. Therefore our portfolio has a lower loading on technology than the current DJIA, which cannot be considered the relevant benchmark because of a technology bias. We deal with all these potential biases by creating benchmarks from exactly the same stocks as the portfolio, so that the benchmarks are affected by the same biases as the portfolio.[6] Subsequently, we analyze all performance on a relative basis.

While a benchmark does not formally enter the portfolio construction model, it is needed to evaluate its performance. By restricting the information used in benchmark construction to the information used in portfolio construction (i.e., the history of stock prices), there are two alternative benchmarks: a price-weighted benchmark (PW), and an equally weighted benchmark (EW).

The first implies no trading as long as the universe of stocks does not change, being self-adjusting to price changes. Therefore, PW is a natural choice as a benchmark for a passive investment strategy. The returns differential between EW and PW will also enter the performance analysis, as a proxy for a value portfolio; by construction, PW places more weight on stocks whose good performance is reflected in their price, so their return difference can be interpreted as a value premium.

For the purposes of principal components analysis, we are particularly interested in the average correlation of the stock returns, as this has a strong influence on the effectiveness of principal components analysis. We find that the average correlation of the daily stock returns from the DJIA set is in the range of 0.3 to 0.4, occasionally going to as low as 0.2. Similar levels of correlation are found in the CAC 40 universe and, as one would expect, lower average correlation for the larger FTSE 100 and S&P 100 indexes. The highest average correlation in stock returns occurs in down volatile markets, such as in 1987, 1990, or 2001–2002. This is a common finding for stock markets.

For the sample period that is common to the four stock universes, 1996–2002, the annual returns and average volatilities of price-weighted indexes of all stocks in each universe are shown in Exhibits 1 and 2. The similarities in the performance of these stock indexes over the period are obvious. Years 1997 and 1999 are associated with the highest returns and lowest volatility, while year 2002 is responsible for the greatest losses and the highest volatility. The CAC 40 appears to be the most volatile and the FTSE 100 the least volatile, mostly because of diversification effects. In the longer DJIA sample, 1987 stands out in terms of returns correlation, volatility, excess kurtosis, and negative skewness, because of the October crash.

EMPIRICAL PROPERTIES OF THE FIRST PRINCIPAL COMPONENT

First, we show that the in-sample properties of the first principal component justify the use of the PC1 portfolio to capture a market factor. Subsequently, we examine the size and the stability of the factor loadings on the first principal component, as this will determine the structure of the PC1 portfolio and the associated transaction costs. Our general conclusions apply to all stock universes, but, given the very long historical data period available and the fact that relatively few stocks are included in the benchmark, these results are best illustrated using the DJIA data set.

We estimate the first principal component of daily stock returns on a rolling sample of 250 observations and compare it with the price-weighted benchmark (PW) of the same stocks. In-sample, the information ratios are very similar, as shown in Exhibit 3, where each point represents the information ratio over the last 250 observations. The main exceptions are the periods 1985-1986 and 1995-1996, when the information ratios of the PC1 portfolio are significantly higher. Additionally, the returns are also highly correlated. The correlation coefficient ranges from 0.70 to 0.98. Lower correlations occur between 1992 and 1996, but most of the time they are still above 0.9. A standard regression of the benchmark returns on the first principal component, estimated over the entire sample, has an R^2 of 0.8. Therefore, we can safely conclude that the first principal component largely captures the market factor.[7]

Of central interest to our analysis are the eigenvectors of the covariance matrix of stock returns, as these will determine the stock weights in the portfolio replicating the first principal component. The eigenvector corresponding to the first principal component represents the sensitivity of each stock to changes in the first principal component, the so-called factor loading. If stock returns were perfectly correlated, the first principal component would capture the entire variation of the system, and the factor loadings would all be equal.

More generally, in a highly but not perfectly correlated system, the factor loadings on the first principal component will be similar but not identical, so that a change in the first principal component generates a nearly parallel shift in the original variables. In this case we can

EXHIBIT 1
Average Returns for Price-Weighted Indexes in DJIA, FTSE 100, CAC 40, and S&P 100 Stock Universes—1996-2002

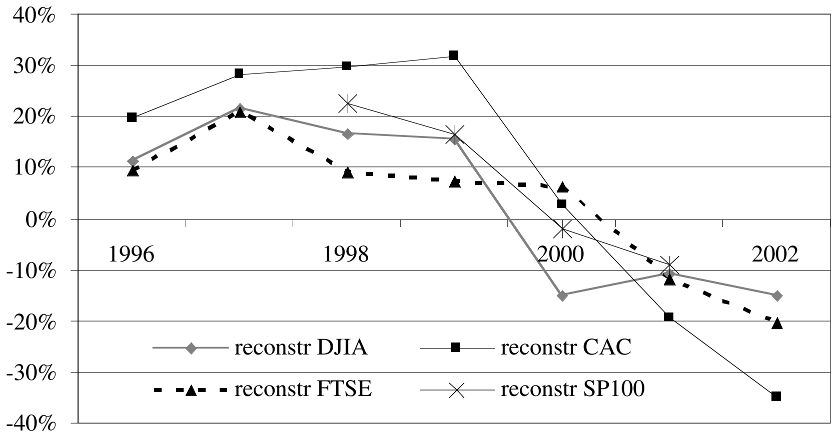


EXHIBIT 2
Average Volatilities for Price-Weighted Indexes in DJIA, FTSE 100, CAC 40, and S&P 100 Stock Universes—1996-2002

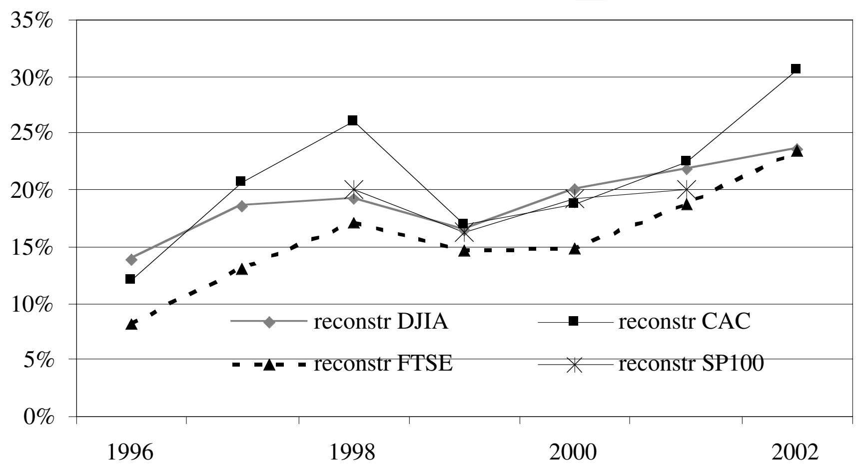
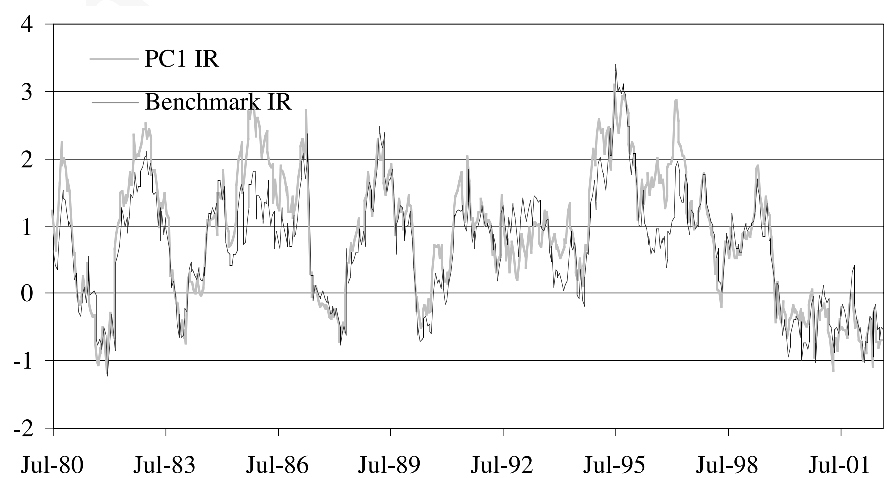
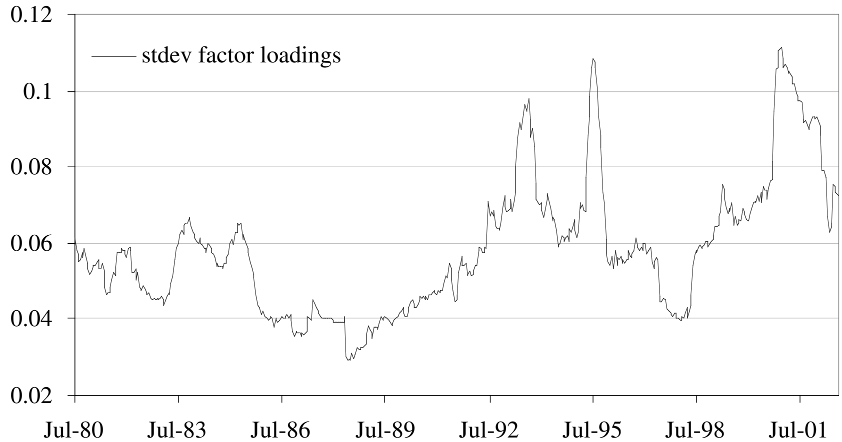


EXHIBIT 3
Information Ratio for PC1 and Benchmark Portfolio



E X H I B I T 4

Cross-Sectional Standard Deviation of Stock Factor Loadings



associate the first principal component with the presence of a common trend in stock returns.

In the DJIA case, the factor loadings on the first principal component are largely in the same range. During periods of high average correlation (e.g., after the 1987 crash), the factor loadings are high and very similar, but more recently they tend to be lower and less similar. This observation is documented in Exhibit 4, which plots the standard deviation of the factor loadings. The similarity of the factor loadings is an important feature of the model, as it allows the construction of balanced portfolios, without extreme exposures to individual stocks. Consequently, we observe that even though there are no short-sale restrictions imposed on the model, short positions occur very rarely.

Dispersion of factor loadings has been used as a measure of herding behavior in recent research in behavioral finance (Hwang and Salmon [2001]), and we return to these implications later when we analyze the model's out-of-sample performance. Apart from the cross-sectional variability of the factor loadings, a very attractive feature is their low time variability. The factor loadings are very stable over time, which in a portfolio construction setting is translated into fewer rebalancing trades and lower transaction costs.

The strength of the common trend is directly related to the total variation explained by the first principal component. In all the stock universes we study, it lies in the range of 20%–40%, in line with other research on this issue (Connor and Korajczyk [1988] and Chan, Karceski, and Lakonishok [1998]). The proportion of variance explained by the first principal component is very close to the average correlation of returns estimated over the same sample. Clearly, the average correlation in the original data is the single most important determinant of the strength of the common trend.

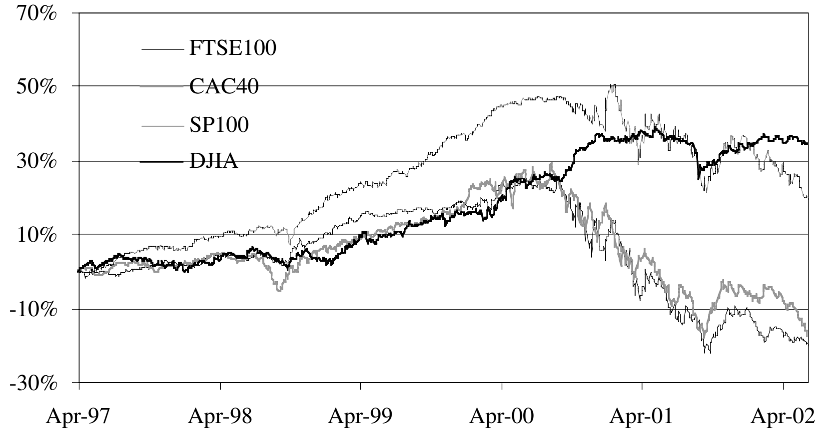
For instance, in the CAC 40 and FTSE 100 universes, the lowest average correlations occur in 1996, the first part of 1997, at the end of 1999, and again in 2000. These periods were relatively calm for the developed stock markets, and correlations are known to be generally higher during more volatile periods. For the same reason, the common trend in stock prices is stronger when the index itself is more volatile.

OUT-OF-SAMPLE PERFORMANCE OF COMMON TREND REPLICATION PORTFOLIO

To investigate the out-of-sample performance of the portfolio replicating the first principal component, the optimization and rebalancing procedure is as follows. At each rebalancing moment, the stock weights are determined from the eigenvectors of the covariance matrix of the stock returns estimated from the most recent 250 observations before the moment of portfolio construction.[8] For the out-of-sample performance assessment, the portfolio constructed in the previous step is left unmanaged for the next ten trading days, and then rebalanced on the basis of the new stock weights from principal components analysis. In between rebalancing, the number of stocks in each portfolio is kept constant and the out-of-sample portfolio performance is recorded.

We use a ten-day rebalancing period, having in mind the institutional investors for which this trading

EXHIBIT 5
Average Cumulative Outperformance in FTSE 100, CAC 40, S&P 100, and DJIA Universes



frequency is standard. The rebalancing frequency could be easily reduced without affecting portfolio performance because portfolio weights are stable over time. To account for transaction costs we assume 20 basis points on each trade value to cover the bid-ask spread and the brokerage commissions.[9]

In each of the S&P 100, FTSE 100, and CAC 40 universes, we select 100 random subsets of stocks, representing 75% of the total number of stocks available. The common sample of data available covers six years, April 1996–June 2002. We also construct the PC1 portfolio in the DJIA stock universe, which, given the longer data sample available (1981–2002), is used as a base case. In each subset, we construct a PW index, an EW index, and a portfolio replicating the first principal component.

To compare the results obtained in the DJIA universe with the ones from the randomly selected subsets of stocks from the S&P 100, the FTSE 100, and the CAC 40 universes, we average the returns of all PC1 portfolios in each universe. Exhibit 5 reports the cumulative average abnormal return of the PC1 portfolio, measured against a PW benchmark, for each market.[11]

One interesting feature of Exhibit 5 is the similarity of the two average return series for the European markets, CAC and FTSE. Both strategies outperform their benchmarks until August 2000, when there is a steady abnormal return. After this date, the abnormal return becomes very volatile and eventually erodes the previous gains.

A relatively similar pattern is also identified for the abnormal return in the S&P 100 and DJIA stock universes. The abnormal returns in the DJIA, however, are much less eroded than those in the S&P 100. This can be due to an increased inertia in the DJIA stocks, and also to the fact that our reduced DJIA universe was not much affected by the technology boom and bust.

We also note a significant difference in the magnitude of returns in the European and U.S. markets. Even before August 2000, the abnormal returns in the S&P 100 and the DJIA are steadier and less volatile than in the European averages. After August 2000, the decline in the average abnormal return in the U.S. markets is less spectacular than in the European markets.

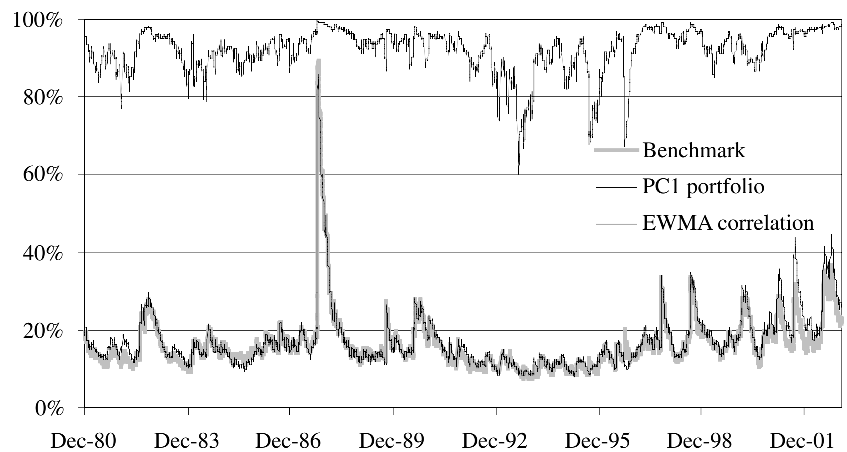
For a more detailed analysis of the performance of the PC1 portfolio, we focus on the results obtained in the DJIA stock universe. The performance statistics for the out-of-sample daily return series of the PC1 portfolio and the two benchmark portfolios are reported in Exhibit 6. The PC1 portfolio outperforms the PW portfolio by an average of over 5 percentage points per year, with only 1 percentage point of extra volatility. EW is also outperformed by the PC1 portfolio by an annual average of almost 3 percentage points. The superior performance is reflected in an information ratio of 0.75 for PC1, compared to 0.50 for PW and 0.63 for EW. The PC1 portfolio returns appear to be marginally closer to normality than the returns on the benchmark portfolios, but all three portfolios have heavy tails and negatively skewed return distributions.

There are other strong similarities among the three portfolios. When out-of-sample returns are analyzed period by period, all portfolios are affected by the major market crises during the period of observation: October 1987, the first Gulf War, the Asian crisis, the burst of the technology bubble, and September 2001. They all show a strong January effect (less evident in the case of the PC1

EXHIBIT 6
Performance Statistics for DJIA Stocks—1981-2002

	PW Benchmark	EW Benchmark	PC1 Portfolio	Excess Return over PW	Excess Return over EW
Annual Return	8.97%	11.34%	14.16%	5.19%	2.70%
Annual Volatility	17.91%	17.84%	18.96%	6.30%	2.59%
Skewness	–1.99	–1.91	–1.54	0.14	0.09
Excess Kurtosis	46.85	40.66	32.45	5.95	18.82
Portfolio Relative Volatility				1.06	1.06
Portfolio Correlation with Benchmark Returns				0.94	0.94
Excess Return Correlation with Benchmark Returns				–0.004	–0.004

EXHIBIT 7
EWMA Volatilities and Correlations in DJIA Universe



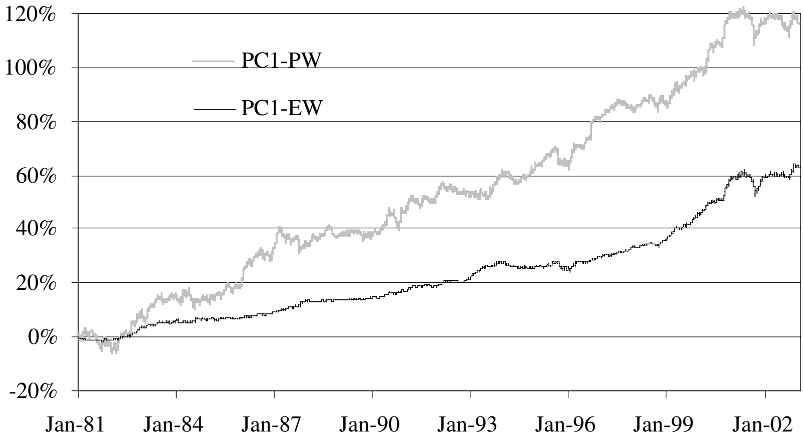
portfolio). The correlation between the PC1 and both benchmark portfolio returns is very high indeed; the PC1 portfolio and the benchmarks have very similar short-term volatility and correlation properties.

The exponentially weighted moving average (EWMA) volatilities and correlation for PC1 and PW with a smoothing parameter of 0.96 are shown in Exhibit 7. The PC1 portfolio is slightly more volatile, especially during the last part of the sample, but closely follows the benchmark volatility. With very few exceptions, the EWMA correlation is high, staying above 0.8 most of the time, and it is particularly high during market crises such as October 1987 or September 2001.

Finally, the transaction costs are almost negligible, amounting to an average of 0.24% per year for implementing the PC1 strategy. As our target is to explain the pure outperformance, i.e., the difference between the PC1 portfolio return and the PW benchmark return, after establishing that the overall profitability of the strategy does not disappear after transaction costs, we look also at portfolio returns before transaction costs.

Considering the PC1 portfolio outperformance with respect to the price-weighted benchmark, the abnormal return can be thought of as produced by a self-financed strategy that at all times is long the PC1 portfolio and short the PW. In this case, the 5.19 percentage point outperformance is associated with an annual volatility of 6.30% (*see Exhibit 6*). The information ratio is 0.82, higher than those of the benchmarks and PC1 portfolio. Moreover, the abnormal return is uncorrelated with the benchmark return and much closer to normality than the benchmark.

E X H I B I T 8
Cumulative Outperformance in DJIA Framework



The fact that the outperformance of the PC1 portfolio is not caused by singular events is evident from Exhibit 8, which shows the cumulative return difference between the PC1 portfolio and the two benchmarks.

EXPLAINING PC1 PORTFOLIO OUTPERFORMANCE

The substantial outperformance of both equally weighted and price-weighted benchmarks by the common trend-replicating portfolio is not the result of data mining.[12] The PC1 portfolio is based on very strong theoretical foundations to capture the common trend in stock returns and is not the result of a blind search through the historical performance of different trading rules. Moreover, it is not betting on a few stocks, having a similar industry and stock diversification to its benchmarks, and it is robust to out-of-sample tests and inclusion of transaction costs.

To explain the performance of the portfolio replicating the first principal component, we develop a simple model based on the relationship between the common trend portfolio and the benchmarks. Given the high correlation between PW and EW, in order to avoid near-multicollinearity, we include in the model only the PW benchmark as a proxy for the market, and, separately, the returns differential between EW and PW as a proxy for a value factor.

A regression model can be estimated using ordinary least squares:

$$\text{PC1\_Return}_t = \alpha + \beta_1(\text{PW\_Return}_t) + \beta_2(\text{EW\_Return} - \text{PW\_Return})_t + \varepsilon_t$$

We have found this specification not robust to heteroscedasticity tests. The pattern in the autocorrelation of squared residuals indicates a GARCH (1, 1) as the alternative, a common specification for stock market index volatility. Additionally, the model does not pass specification error tests, indicating a possible non-linear relationship between PC1 portfolio returns and one of the explanatory variables. These specification tests improve when squared returns on the PW benchmark are included as follows:

$$\begin{aligned} \text{PC1\_Return}_t &= \acute{a} + \hat{a}_1 (\text{PW\_Return}_t) + \hat{a}_2(\text{EW\_Return} — \text{PW\_Return})_t + \hat{a}_3 (\text{PW\_return}_t^2) + \mathring{a}_1 \\ \varepsilon_1 | \text{I}_t &\cong \text{N}(0, \sigma_t^2) \\ \acute{o}_t^2 &= \grave{u} + \acute{a}_v \mathring{a}_{t\text{-}1}^2 + \hat{a}_v \acute{o}_{t\text{-}1}^2 \end{aligned} \quad (3)$$

When we apply this model to the CAC 40, the FTSE 100, the S&P 100, and the DJIA stock universes, we find the intercept term not statistically significant in most cases. Thus, the model provides a complete decomposition of the common trend portfolio benchmark outperformance into three risk factor premiums:

1. If the coefficient of the price-weighted benchmark returns, which measures the portfolio's sensitivity to the market factor, is greater than unity, part of the portfolio outperformance can be attributed to a higher loading on the market risk factor.
2. When a positive relationship between the portfolio return and the value factor proxy is found,

this will indicate that part of the outperformance is due to a higher loading on value stocks.
3. Finally, the finding of a positive and significant coefficient of the squared price-weighted benchmark returns can be interpreted as the portfolio sensitivity to a volatility factor, proxied by the squared market returns.

The portfolio premium on each factor is defined as the product of the portfolio sensitivity to that factor and the factor premium. If we combine the market premium earned by the PC1 portfolio with a volatility premium, the return differential between the two and the price-weighted benchmark has a straddle pattern; the PC1 portfolio outperforms large negative and large positive benchmark returns, and marginally underperforms small negative market returns. This feature of the strategy is important, as it lets investors reduce the portfolio exposure to negative market circumstances, while increasing the exposure to positive ones. From this point of view, the strategy will act like a benchmark enhancer.

The average contributions of the risk factors to portfolio outperformance, estimated according to Equation (3), are presented in Exhibit 9 over April 1997–June 2002. The first observation is that for all stock universes the volatility premium dominates the other two. These results come as no surprise, considering the high volatility in these equity markets during most of this period. Also, there is a significant market premium. The contribution of the value premium is either negative, for the CAC 40 and the S&P 100, or marginally positive, in the case of the FTSE 100. The only universe that exhibits a significantly positive value premium for the period is the DJIA.

The similarity in the performance of portfolios constructed in different stock universes can be interpreted as evidence of common trends within the international stock markets. Usually, such evidence has been produced as a result of examination of the properties of different market indexes or groups of stocks, such as cointegration or correlation in different market circumstances. The evidence of similarities in the performance of a strategy, as a dynamic combination of stocks, in different markets is equally relevant for the hypothesis of common movements, even if indirect. Moreover, we have shown that there are also significant similarities in the contribution of the risk factors to the strategy outperformance.

EXHIBIT 9
Contribution of Risk Factors to Strategy Outperformance in FTSE 100, CAC 40, S&P 100, and DJIA Universes

	Value	Market	Volatility
FTSE 100	0.14%	0.85%	2.19%
CAC 40	-0.40%	0.89%	1.72%
S&P 100	-0.82%	0.88%	2.21%
DJIA	1.57%	0.47%	1.42%

The full estimation results for Equation (3) in the DJIA universe are reported in Exhibit 10.[13] The very high R^2, above 0.98, comes as no surprise, considering the strong correlation between the portfolio return and its benchmarks. All coefficients, except for the mean regression intercept, are positive and highly significant at the 1% significance level. The variance regression model estimates show an almost integrated GARCH model for the variance of the PC1 portfolio, with persistence coefficient (0.96) and reaction coefficient (0.04) in the usual ranges for stock market volatility during this sample period.

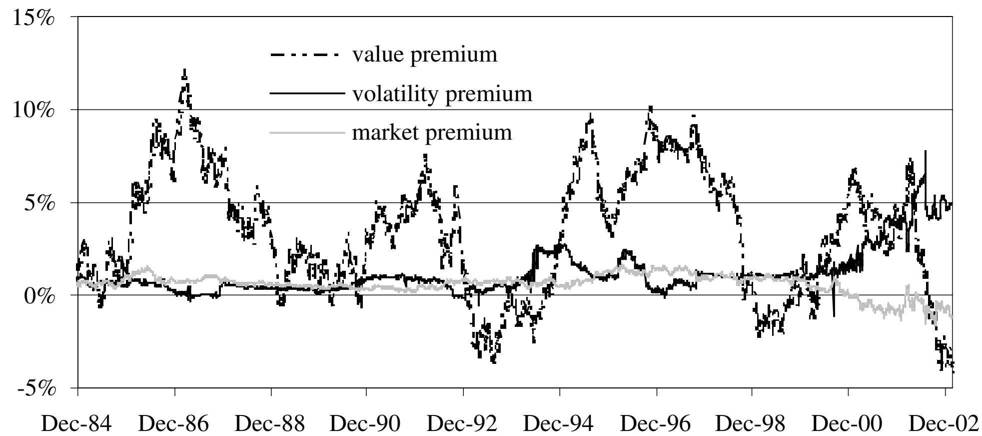
Over the 1981-2002 period, the contribution of the three sources to the total outperformance of the PC1 portfolio is as follows: market premium 11%, value premium 60%, and volatility premium 29%. Yet this distribution is

EXHIBIT 10
Estimated Coefficients of Equation (3) in DJIA Universe

	α	β_1	β_2	β_3	ω	α_v	β_v
Coefficient	-4.6E-09	1.04432	1.07232	0.43170	2.29E-09	0.03765	0.96148
Std Error	1.18E-05	0.00108	0.00311	0.01675	5.47E-10	0.00279	0.00278
t-Statistic	-0.00039	964.390	344.251	25.7725	4.178975	13.4734	345.436
P-Value	0.99	0.00	0.00	0.00	0.00	0.00	0.00

EXHIBIT 11

Time Distribution of Outperformance Sources in DJIA Universe



far from stationary, as shown in Exhibit 11. The plot shows the contributions of the three outperformance sources estimated using Equation (3) on a rolling window of 500 daily observations. Each premium is computed as the portfolio sensitivity to the factor times the annualized mean factor premium over the estimation sample.

The *market premium* makes the most stable but the smallest contribution to the portfolio outperformance, becoming negative from 2001 as the stock market generally declined. The PC1 portfolio, with a high market beta, outperforms in down markets.

The *value premium* accounts for the greatest part of the portfolio outperformance during most of the 1980s and 1990s. This finding is consistent with abnormal returns generated by a mean reversion mechanism, as we argue below. The value premium does fall sharply and become negative around the time of the October 1987 crash, after the first Gulf War, during the Asian crisis, and again at the end of the sample. An explanation for this change is that the normal mean reversion cycle is broken around the time of market crises, because investor behavior changes significantly.

The *volatility premium* increased markedly during 2001 and 2002, given the increased stock market turbulence of the last few years. It accounts for 70% of the total outperformance by the end of the sample.

BEHAVIORAL MECHANISM FOR PC1 PORTFOLIO OUTPERFORMANCE

Having identified the sources of portfolio outperformance, we can demonstrate a mechanism through which part of this outperformance might be achieved. The stock weights in the PC1 portfolio have been chosen to maximize the portfolio variance, subject to the constraint of a unit norm for the factor loadings. Since portfolio variance increases with both individual asset variance and the covariance between assets, the portfolio will overweight (relative to the PW benchmark) stocks that have higher volatility over the estimation period and that are also highly correlated as a group. Separately, the PW benchmark is underweighting stocks that have recently declined.

Now, if it does hold true that markets tend to be more turbulent after a large price fall than after a similar price increase (a leverage effect commonly identified in stock markets, as in Black [1976], Christie [1982], French, Schwert, and Stambaugh [1987]), the same group of stocks will be impacted through the overweighting of volatile correlated stocks in the PC1 portfolio and the underweighting of declining stocks in the PW benchmark.

From this perspective, the outperformance of the PC1 portfolio must be due to a mean reversion in stock returns over the one-year estimation period used for the portfolio. The portfolio overweights stocks that have recently declined in price, relative to the benchmark, so the relative profit on the portfolio has to be the result of a subsequent rise in price of these stocks.

The hypothesis that mean reversion takes place over a period of one year is supported by the fact that when the PC1 estimation sample is reduced, the in-sample outperformance of the first principal component with respect to the PW benchmark disappears.

These results are in line with research on the short-term momentum and long-term reversals frequently identified in stock returns. For example, De Bondt and Thaler [1985], Lo and MacKinlay [1988], Poterba and Summers [1988], and Jegadeesh and Titman [1993] identify positive autocorrelation in stock returns at intervals of less than one year and negative autocorrelation at longer intervals.

In behavioral finance, two explanations are usually proffered for long-term reversals and short-term momentum in stock markets. The first explanation focuses on relatively volatile stocks, which capture the attention of noise traders for whom they are the best buy candidates (Odean [1999]).[14]

The trading behavior of noise traders creates an upward price pressure on these volatile stocks, forcing mean reversion when their high volatility is associated with a recent decline in price. The same explanation is not applicable to a selling decision, creating symmetrically downward price pressure on volatile stocks, because the range of choice in a selling decision is usually limited to the stocks already held (Barber and Odean [2002]). Additionally, we note that volatile stocks that have recently experienced a price decline also qualify as value stocks, so this explains the value premium previously observed.

A second behavioral explanation of the short-term momentum followed by mean reversion has been provided by De Long et al. [1990b], Lakonishok, Shleifer, and Vishny [1994], and Shleifer and Vishny [1997]. This explanation is based on investor sentiment, overreaction, and excessive optimism or pessimism. The occurrence of some bad news regarding one stock creates some initial excess volatility, and, according to these models, some investors will become pessimistic about that stock and start selling. If there is positive feedback in the market, more selling will follow, and the selling pressure will drive the price below its fundamental level.

The arbitrageurs (sometimes called smart money, or rational investors) will not take positions against the mispricing either because 1) the mispricing is too minor to justify arbitrage after transaction costs, or 2) there is no appropriate replacement available for that stock, so the fundamental risk cannot be hedged away, or 3) there is a noise trader risk arising from positive feedback, in that excessive investor pessimism will drive the price even farther down over the short term. In the presence of positive feedback, De Long et al. [1990a] show that the arbitrageurs will initially join the noise traders in selling, in order to close their positions when the mispricing has become even greater. This type of behavior justifies both short-term momentum and longer-term mean reversion.

We also find there is a connection between the abnormal returns generated by the PC1 portfolio and another behavioral phenomenon that is well documented in stock markets—investor herding. Herding behavior is measured by a decline in the cross-sectional standard deviation of the factor loadings. We find that the more intense the herding, the higher the abnormal returns generated by the PC1 portfolio.

The cross-sectional distribution of stock returns as an indication of herding was first introduced by Christie and Huang [1995] in the form of a cross-sectional standard deviation of individual stock returns during large price changes. Hwang and Salmon [2001] build on this idea but instead advocate the use of a standardized standard deviation of PCA factor loadings to measure the degree of herding. Their measure has the advantage of capturing intentional herding toward a given factor, such as the market factor, rather than spurious herding during market crises.

Following Hwang and Salmon [2001], we assume that the standard deviation of the factor loadings captures the intentional herding of investors toward the first principal component of the stocks, or their common trend. An intense herding of investors toward the common trend of stocks should diminish the differences in the individual stock loadings on the first principal component. Therefore, we interpret a low standard deviation of the factor loadings as an indication of herding.

In Exhibit 4, for instance, we see that more intense herding appears to happen before 1993, and then again before 1998, which supports the findings in Hwang and Salmon [2001] that this type of herding occurs especially during quiet periods for the market. During the market crises of the last five years, herding behavior appears to be significantly reduced.

Given these interpretations of mean-reverting behavior, an intense herding toward the first principal component, indicated by a sharp reduction in the standard deviation of the factor loadings, should enhance and speed up the mean reversion. Therefore, the standard deviation of the factor loadings should be negatively related to the outperformance of the PC1 portfolio.

Indeed, the correlation between the standard deviation of the factor loadings and the abnormal return, estimated over all non-overlapping subsamples of 120 observations, is negative (-0.33) and significant at the 5% level. Of the three different sources of outperformance, this measure of herding is most closely related to the value premium. The smaller the standard deviation of the factor loadings, the more similar the PC1 portfolio will be to an equally weighted portfolio, and consequently the higher the value premium.

SUMMARY AND CONCLUSIONS

We have proposed a portfolio construction model based on the principal components analysis of stock returns. As opposed to traditional approaches to indexing, which aim to replicate the performance of a standard benchmark, our model is based on the replication of only the common trend of the stocks included in that benchmark.

The model identifies of all possible combinations of stocks with unit norm weights the portfolio that captures the greatest part of the total joint variation of stock returns. By doing so, the strategy manages to filter out a significant amount of the noise in stock returns, which facilitates the replication task considerably. On these grounds, the PC1 portfolio structure turns out to be very stable over time, requiring only a minimal amount of rebalancing, which results in negligible transaction costs, amounting to less than __% per year.

Moreover, we have shown that over a long period of time, the PC1 portfolio, while highly correlated with its benchmarks, has significantly outperformed them. The sources of outperformance are shown to be a market premium, a value premium, and a volatility premium. The straddle pattern created by the volatility premium is particularly appealing for investors, reducing the exposure to large negative benchmark returns and increasing the exposure to positive benchmark returns.

One mechanism explaining the outperformance is mean reversion in returns for the stocks that are overweighted by the portfolio, that is, stocks that have had higher volatility and have also been highly correlated as a group, during the portfolio calibration period. We point out two behavioral phenomena that could be driving the mean reversion for these stocks: the attention-capturing effect and investor overreaction, both of them resulting in different forms of herding behavior. Indeed, we find a close relationship between the abnormal return and a measure of investor herding toward the market factor.

The distribution in time of the outperformance sources has been shown to evolve from a value-dominated outperformance toward an increased volatility premium associated with the volatile years toward the end of our data sample. This finding supports the behavioral mechanisms thought to be driving the mean reversion in stock returns, to the extent that during volatile periods investors tend to herd less, and this prevents mean reversion from taking place.

Finally, we find a common pattern in the performance of the strategy when applied to European and U.S. stock markets. There is a high correlation between the strategy results for the two European indexes, and a high correlation between the results for the two U.S. indexes, but a lower correlation between the results for the European and U.S. indexes. Despite significant similarities in the relative contribution of the risk factors to the strategy outperformance, the differences in the patterns of the U.S. and European results, however small, do represent a potential for diversification. Extending the analysis to other stock markets less correlated with the U.S. and European ones could uncover even better diversification opportunities.

ENDNOTES

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[1]We use the term *market* to denote the specific universe of stocks targeted by the passive investment strategy, which can be anything between a selection of stocks and the true *market portfolio*, representing all assets.

[2]Outperformance is defined as the difference between the factor-mimicking portfolio returns and the returns of a price-weighted benchmark, reconstructed from the same stocks as the portfolio.

[3]Eigenvectors are not unique, so it is standard to impose the orthonormal constraint. A more natural constraint in a portfolio construction framework would be to have the sum of the eigenvectors, rather than the sum of their squares, equal to one, but this does not ensure the balanced portfolio structure that is essential for indexing. To avoid large exposures to individual stocks, we keep the unit length constraint for the eigenvectors, and then normalize them to sum up to one.

[4]Note that often the original stationary variables are standardized to have zero mean and unit variance before the principal components analysis—i.e., the eigenvectors of the correlation matrix are used to construct the principal components, rather than the eigenvectors of the covariance matrix. This ensures that the variable with the highest volatility does not dominate the first principal component. In a realistic portfolio construction setting, the assumption of equal volatilities for all assets is not feasible. Such an assumption would result in con-

structing the portfolio solely on the correlation structure of the assets, rather than the complete covariance structure of the data. Therefore, for the purpose of our model, we do not standardize the stock returns.

[5]The unit norm constraint ensures a balanced portfolio structure, without high exposures to individual stocks. This constraint can also be interpreted as a Bayesian approach to limiting the effect of outliers in the historic stock returns—a heavy weight on an individual stock results when the stock has a very high in-sample volatility, but this could simply be due to measurement errors or single outliers (Jagannathan and Ma [2002]).

[6]The alternative is to include in the portfolio at time *t* the stocks that were in the benchmark at time *t*, but this would necessitate a complex dynamic back-test procedure, and the underlying principle is the same (that the stocks in the benchmark are the same as the stocks in the portfolio). Actually, over the entire data sample the price-weighted benchmark of the DJIA stocks had a cumulative return of 203% and the actual DJIA returned 215%. Therefore, the survivorship bias due to the difference in technology loadings is in fact negative.

[7]The term *market* refers to the specific universe of stocks included in our analysis. As this universe increases, it will converge to the true *market portfolio*.

[8]The "rolling sample" PCA raises the issue of consistent identification of the factor loadings because the choice of the sign of the eigenvectors is arbitrary. Choosing a particular normalization is not relevant if the estimation of the principal components is performed over the entire data sample. When the optimization is performed over a rolling sample, in order to have consistent principal component estimates from successive estimations, one needs to ensure that the same normalization is used throughout the entire data sample. To this end, following Chan, Karceski, and Lakonishok [1998], we impose an additional restriction; i.e., the first principal component needs to be positively correlated with the price-weighted portfolio of all stocks.

[9]We do not account for potential tax implications for individual investors, assuming that the strategy is designed primarily for institutional investors.

[10]There is always a trade-off between the diversity of portfolios within one universe and the number of stocks selected in each portfolio. 75% of all stocks in each portfolio ensures a relative balance of the two. In the CAC universe, the subset has 23 stocks; in the FTSE universe there are 52 stocks; and in the SP universe there are 75 stocks.

[11]We define the abnormal return as the difference between the factor-mimicking portfolio returns and the returns of a price-weighted benchmark, reconstructed from the same stocks as the portfolio.

[12]Any strategy that outperforms in the DJIA stock universe should lead one to question whether there is any connection with the famous "dogs of the Dow" or "fool's four" value strategies, which pick a small number of stocks from the DJIA (ten and four, respectively), and rebalance as rarely as once a year. They are known to be a classic case of data mining, that is, an extensive search through a large number of trading strategies for those that have historically outperformed the benchmark. Apart from low diversification and increased volatility, these strategies have been shown not robust to out-of-sample tests and associated transaction costs (McQueen and Thorley [1999] and Hirschey [2000]).

[13]This version of the model passes the autocorrelation and ARCH tests, having slightly non-normal residuals, probably due to the presence of outliers. The information criteria clearly favor this specification as compared to the initial one.

[14]Noise traders are usually defined in the literature as not fully rational investors who make investment decisions based on beliefs or sentiments that are not fully justified by fundamental news, or that are subject to systematic biases.

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