9 Hedging the Risk of an Energy[§] Futures Portfolio

Carol Alexander

This chapter considers a hedging problem for a trader in futures on crude oil, heating oil and unleaded gasoline and on the crack spreads based on these energy commodities. We first explain how the trader can map his current position to use constant maturity futures as risk factors. This has many advantages over using spot price or prompt futures prices plus discount rates as risk factors. Then we show how the trader can quantify his key risk factors, assess the risk of his portfolio and determine the most cost-effective hedging strategies.

The outline of this chapter is as follows. Section 9.1 explains the risk factor mapping process, and the advantages of using constant maturity futures as risk factors; Section 9.2 describes the portfolio to be hedged and Section 9.3 explains how principal component analysis is applied to reduce the dimension of the risk factor space and to isolate the key sources of risk; Section 9.4 assesses the risk of the portfolio and describes how best to hedge this risk; and Section 9.5 concludes.

9.1 MAPPING PORTFOLIOS TO CONSTANT MATURITY FUTURES

Constant maturity futures are not traded instruments. However, a time series of constant maturity commodity futures can be obtained by *concatenation* of adjacent futures prices. For instance, a time series for a constant maturity 1-month futures price can be obtained by taking the prompt futures with expiry less than or equal to 1 month and the next futures with expiry greater than 1 month and linearly interpolating between the two prices. So if the prompt futures contract with price P_1 has maturity $T_1 \leqslant 1$ month and the futures contract with price P_2 is the next to expire with maturity $T_2 > 1$ month, and T_1 and T_2 are measured in years, then the 1-month futures price is

$$P = \frac{\left(T_2 - \frac{1}{12}\right) P_1 + \left(\frac{1}{12} - T_1\right) P_2}{(T_2 - T_1)}.$$

ICMA Centre, University of Reading

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Carol Alexander, Professor of Financial Risk Management and Director of Research, ICMA Centre, University of Reading, UK; c.alexander@icmacentre.rdg.ac.uk.

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For instance, suppose we wish to construct a 3-month futures series when the maturity dates are 16 March, 16 June, 16 September and 16 December. On 1 September we use the September and the December contracts, with prices P_1 and P_2 respectively. The number of days between the September contract expiry date and our 3-month expiry date (which is 1 December on 1 September) is 76 and the time interval between the December contract expiry date and our expiry date is 15 days. Hence, the concatenated price is

$$\frac{15 \times P_1 + 76 \times P_2}{91}.$$

We can continue to use the September and December contract prices in the construction of the 1-month futures price. For instance, on 12 September, our 3-month expiry date is 12 December, so the concatenated price would be

$$\frac{4 \times P_1 + 87 \times P_2}{91},$$

where P_1 and P_2 are now the prices of the September and December futures contracts on 12 September.

Since prices can behave oddly a few days before expiry, we should drop the September contract from our calculations after 12 September, and instead take the 3-month maturity contract price to be equal to the December contract price for a few days. However, on 17 September we can start using linear interpolation between the December and March futures prices as above, but now with the December contract being the shorter one. As time moves on we decrease the weight of the December price in our calculation of the concatenated futures price and increase the weight on the March futures price.

The main advantage of using constant maturity futures as risk factors for commodity portfolios is that the use of spot prices plus discount rates as risk factors assumes futures are always at their fair price, and thus ignores the basis risk due to fluctuations in carry costs and convenience yields, as well as the variation of the market price of the futures within its no arbitrage range. All these sources of uncertainty in the basis are considerable: carry costs due to storage, insurance and transportation are difficult to measure precisely; convenience yields are intangible and even more difficult to assess; and since the spot cannot be shorted the no arbitrage range has no lower bound, whence substantial decoupling of spot and futures prices is often evident when demand surges or supply drops for reasons beyond the control of the market participants.

Constant maturity futures provide a long time series of futures prices that can be used to assess the market characteristics. These characteristics vary considerably from market to market. Prices are determined by unpredictable demand and supply factors such as the weather and the economic climate. For instance, the weather affects the supply of corn and the demand for gas and the outbreak of war affects the price of oil. But prices may also be affected by speculative trading, and the "herding" behaviour of speculative investors can lead to prolonged price trends in futures prices that have nothing to do with demand and supply of the actual commodity.

The process of mapping a portfolio to constant maturity futures is very similar to the process of mapping a cash flow to fixed maturity interest rates. This can be done in a present value and volatility invariant manner as the examples below will demonstrate. The methodology is a simple adaptation of the cash flow mapping methods that are described for interest rate sensitive portfolios in Alexander (2008) on pages 332–37.

Example 1 Mapping commodity forward positions to constant maturity futures

Suppose we have just two forward positions on crude oil: a long position with present value \$2 million in 1 month and 10 days and a short position with present value −\$1 million in 1 month and 20 days. How should we map these to equivalent positions on constant maturity crude oil futures at the 1-month and 2-month maturity?

Solution

Suppose there are 30 days in a month. We could simply use linear interpolation and map $2/3 \times 2 - 1/3 \times 1 = \$1\text{m}$ to the 1-month vertex and $1/3 \times 2 - 2/3 \times 1 = 0$ to the 2-month vertex. But this will change the volatility of the portfolio. Instead we can use a present value and volatility invariant map, depicted in Fig 9.1. For this we need to know the volatilities of the 1-month and 2-month futures and their correlation – suppose the volatilities are 30 % and 27 % as shown and the correlation is 0.95. Using linear interpolation on the variances we infer the volatilities of 29.03 % for the 1 month 10 day position and 28.04 % for the 1 month 20 day position.

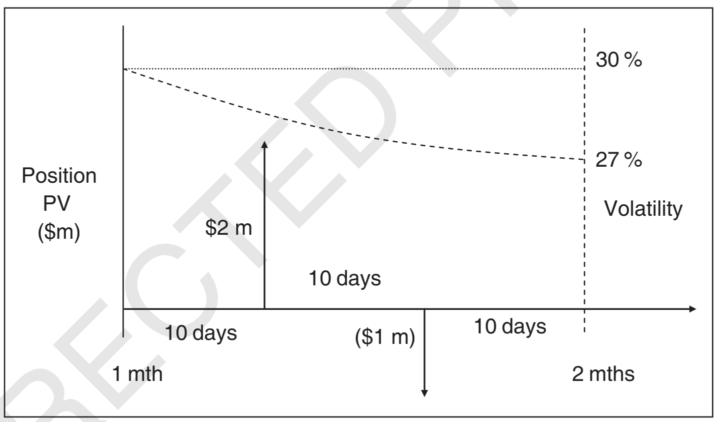

Figure 9.1 A volatility invariant commodity futures or forwards mapping

For the mapped position to have the same volatility as the original position, the proportion x of the long position of \$2 million at 1 month 10 days that is mapped to the 1-month future must satisfy

$$29.03^2 = 30^2 \times x^2 + 27^2 \times (1 - x)^2 + 2 \times 0.95 \times 30 \times 27 \times x \times (1 - x).$$

This quadratic equation has one solution between 0 and 1, i.e. $x = 0.762092$. Similarly, the proportion y of the short position of \$1 million at 1 month 20 days that is mapped to the 1-month future must satisfy

$$28.04^2 = 30^2 \times y^2 + 27^2 \times (1 - y)^2 + 2 \times 0.95 \times 30 \times 27 \times y \times (1 - y),$$

and solving this gives $y = 0.464239$. Thus in total we must map

$$0.762092 \times \$2\,000\,000 - 0.464239 \times \$1\,000\,000 = \$1\,059\,945$$

to the 1-month future, and

$$0.237908 \times \$2\,000\,000 - 0.535761 \times \$1\,000\,000 = -\$59\,945$$

to the 2-month future. This way the present value and the volatility of the mapped position is the same as the volatility of the unmapped position.

Example 2 Determining the number of contracts

Suppose the mapping in Example 1 was performed on a day when the prices of the crude oil forwards and futures were as shown in Table 9.1. Each forward contract is for 100 barrels and each futures contract is for 1000 barrels of crude oil. How many contracts are in the mapped position?

Table 9.1 Prices of crude oil forwards and futures ($ per barrel)

30 days	40 days	50 days	60 days
98.35	99.01	100	100.57

Solution

The solution is summarized in Table 9.2. The second row of the table shows the unmapped and mapped value of the positions, the number of barrels is this position value divided by the price per barrel and the number contracts is the number of barrels divided by the number of barrels in each contract (1000 for the futures and 100 for the forwards).

Table 9.2 Positions on crude oil forwards and futures

Maturity	30 days	40 days	50 days	60 days
Price	98.35	99.01	100	100.57
Position Value	1 059 945	2 000 000	−1 000 000	−59,945
Number of Barrels	10 777.28	20 200	−10 000	−596.05
Number of Contracts	10.777	202	−100	−0.5961

Of course, there is no need to round the resulting number of futures contracts to the nearest integer, since we are only mapping the portfolio to these non-traded risk factors.

9.2 THE PORTFOLIO AND ITS KEY RISK FACTORS

Suppose that on 1 August 2006 a trader in energy futures holds long and short positions that have been mapped to constant maturity futures as shown in Table 9.3. Each futures contract is for 1000 barrels and the minus sign indicates a short position. Note that these positions could result from both straight futures trades and from positions on the two crack spread futures, i.e. unleaded gasoline−crude oil and heating oil−crude oil.

Figures 9.2−9.4 show how the daily prices of the constant maturity futures on all three products have evolved over a very long period. All prices spiked before the outbreak of

Table 9.3 Number of futures contracts in an energy futures trading book

Maturity (months)	Crude oil	Heating oil	Unleaded gasoline
1	−100	70	20
2	180	−60	−60
3	−300	150	100
4	−400	200	250
5	250	−180	−100
6	−100	100	30

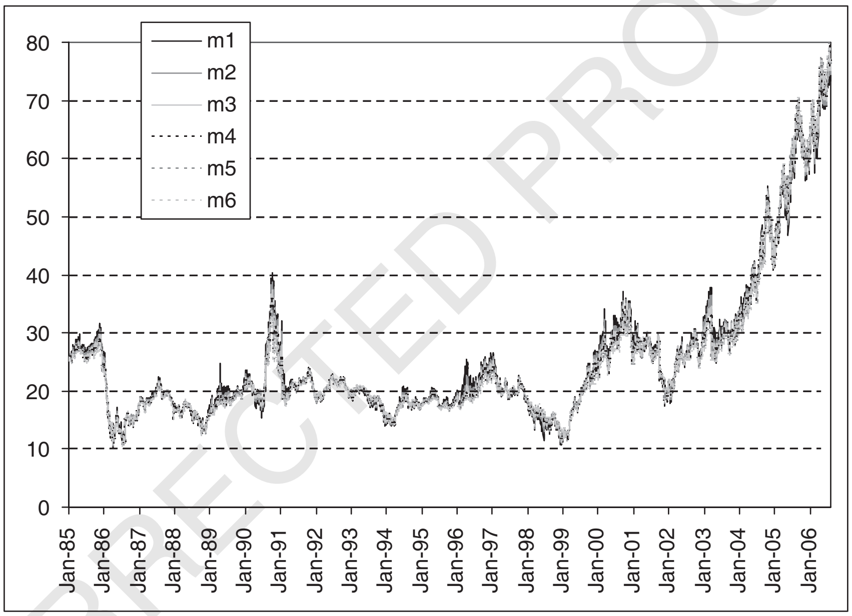

Figure 9.2 NYMEX WTI crude oil constant maturity futures prices

the Gulf war in 1991, and since the war in Iraq in 2003 prices have risen tremendously. For instance, the crude oil futures prices rose from around $20 per barrel to nearly $80 per barrel in August 2006 and the prices of the futures on the refined products rose even more. The daily price fluctuations of futures of different maturities on each product are always very highly correlated, as are those on futures of different products.

We do not need to use 30 years of data for the portfolio risk analysis; in fact looking back into the 1990s and beyond may give misleading results since energy markets have become much more volatile during the last few years. But we do need fairly high frequency data, because the trader needs to understand his short term risks. So we shall use daily data between 2 January 1999 and August 2006. ~~In the spreadsheet for this case study w~~e first calculate the average correlation of daily returns on each of the futures over the sample period. The correlation matrix is too large to be reported in the text, but Table 9.4 shows some of these correlations.

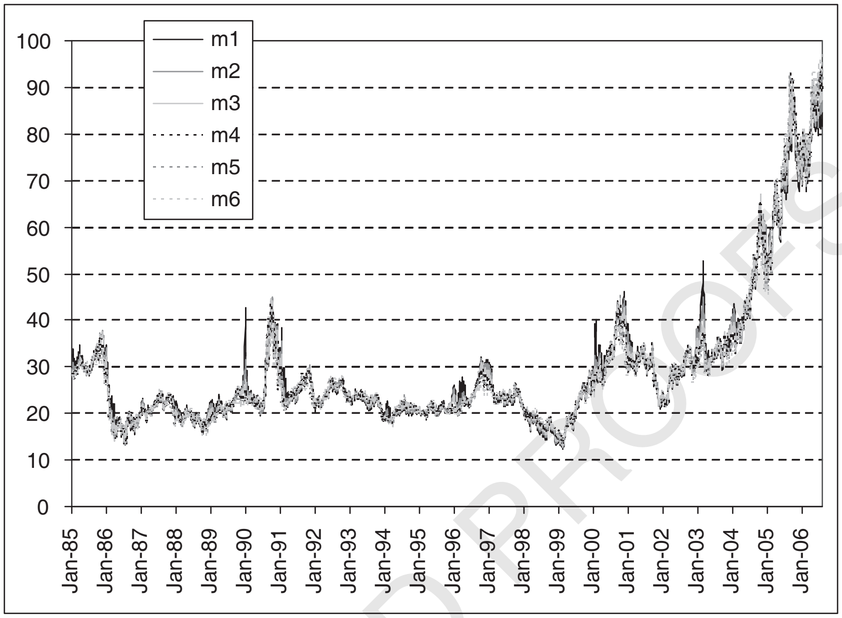

Figure 9.3 NYMEX heating oil constant maturity futures prices

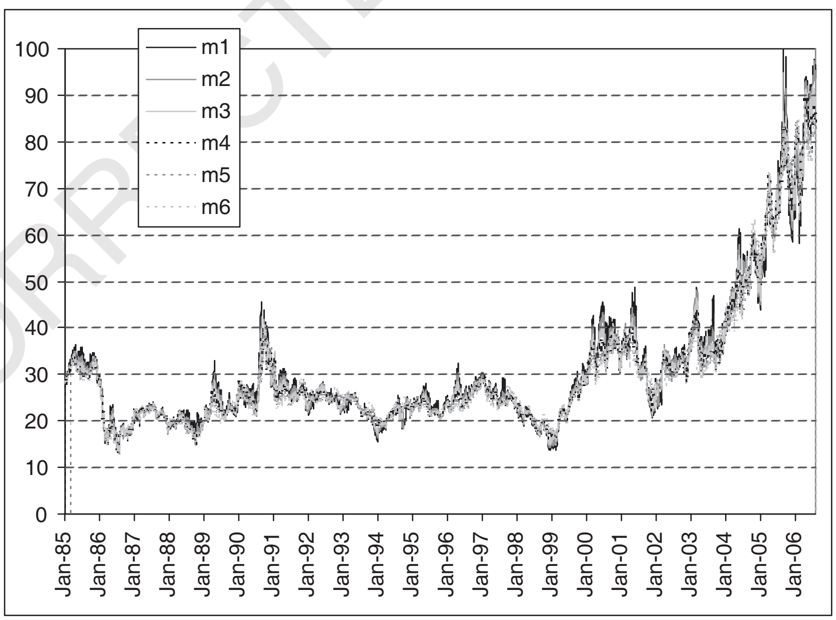

Figure 9.4 NYMEX unleaded gasoline constant maturity futures prices

Table 9.4 Daily correlations of futures prices at selected maturities

Correlatons	m1–m2	m2–m3	m3–m4	m4–m5	m5–m6
Crude oil (CO)	0.972	0.994	0.997	0.998	0.998
Heating oil (HO)	0.946	0.986	0.986	0.991	0.991
Unleaded gasoline (UL)	0.918	0.951	0.951	0.947	0.949
Cross-correlations	m1	m2	m3	m4	m6
HO–CO	0.776	0.878	0.888	0.895	0.894
UL–CO	0.728	0.839	0.857	0.849	0.852
HO–UL	0.695	0.804	0.815	0.812	0.811

The crude oil futures behave like a typical term structure, with the correlation being higher at the long end and decreasing as the maturity gap between the futures increases. All correlations are very high indeed. The same comments apply to heating oil and unleaded gasoline futures, and although their term structures are a little less highly correlated than crude oil there is still a very high degree of correlation. The lower part of Table 9.4 shows that the cross-correlations between futures on different products are lower than the correlations of futures on one of the products, but they are still very high. Note that the 1-month futures tend to have slightly lower correlations than futures of 2 months' maturity and longer.

9.3 IDENTIFYING THE KEY RISK FACTORS

We have a total of 18 risk factors. But since they are so highly correlated we should perform a principal component analysis (PCA) to reduce the dimension of the risk factor space.[1] ~~In the spreadsheet~~ we apply PCA to the entire system of 18 risk factors. ~~This way~~ the principal component risk factors capture correlated movements across futures on different commodities, as well as within futures on the same commodity. The PCA may be applied to either the correlation or the covariance matrix of returns, with the latter accounting for any difference between the risk factor volatilities. In each commodity the 1-month futures have noticeably higher volatilities than the other futures, so we shall perform the PCA on the covariance matrix.

The results for the first four components from the PCA on the covariance matrix are displayed in Table 9.5. The entries in Table 9.5 are the eigenvectors corresponding to the first, second, third and fourth largest eigenvalues. The percentage of the total variation that is explained by each eigenvalue is shown in the first row of the table. Examining the eigenvalues and eigenvectors we deduce that, between January 1999 and August 2006:

- 86 % of the historical variations were a similar and simultaneous shift and tilt in all three term structures;
- 5 % of the historical variations were when the crude oil and unleaded gasoline futures shift and tilt in opposite directions and the heating oil futures term structure tilts changes convexity;

[1] See Alexander (2008) Section I.2.6 ~~and~~ Chapter II.2 for further details of principal component analysis.

Table 9.5 Results of PCA on the futures returns covariance matrix

	86%	5%	3%	2%
WTI m1	0.2594	0.1326	0.3522	0.0562
WTI m2	0.2448	0.1183	0.3055	0.0387
WTI m3	0.2284	0.1159	0.2908	0.0303
WTI m4	0.2157	0.1133	0.2802	0.0255
WTI m5	0.2053	0.1112	0.2697	0.0225
WTI m6	0.1965	0.1086	0.2587	0.0183
HO m1	0.2750	0.2245	−0.5156	−0.2024
HO m2	0.2629	0.2342	−0.3045	−0.0457
HO m3	0.2449	0.2242	−0.2283	0.0654
HO m4	0.2316	0.1979	−0.1618	0.1777
HO m5	0.2210	0.1611	−0.1158	0.2479
HO m6	0.2126	0.1120	−0.0772	0.2676
UL m1	0.2835	−0.6028	−0.1512	0.5121
UL m2	0.2630	−0.3950	−0.0172	0.0412
UL m3	0.2390	−0.2952	0.0183	−0.2175
UL m4	0.2210	−0.2249	0.0066	−0.3559
UL m5	0.2094	−0.1452	0.0018	−0.4224
UL m6	0.2039	−0.0810	−0.0041	−0.4057

- 3 % of the historical variations were when the crude oil and heating oil futures term structures shift and tilt in opposite directions and the unleaded gasoline futures remain static except at the very short end;
- 2 % of the historical variations were when the crude oil futures remain almost static, and the heating oil futures and unleaded gasoline futures tilt in opposite directions.

The first four principal components are time series that represent the four key risk factors for *any* portfolio on these oil futures. The common trend principal component risk factor is much the most important, since 86 % of the historical variation in these futures was due to movements of this type. Taking the first four components together captures 96 % of the historical variations in these energy futures since January 1999.

9.4 HEDGING THE PORTFOLIO RISK

Figure 9.5 shows a reconstructed price series for the portfolio, holding the positions constant at the values shown in Table 9.3 and revaluing the portfolio using the historical prices of the constant maturity futures. Since the portfolio has short positions its value could become negative, hence we base our analysis on the portfolio P&L and not on portfolio returns.

The current value of the portfolio is \$10.025 million and its historical P&L volatility based on the reconstructed price series is extremely high, at over \$3.5 million per annum. The 1 % 1-day historical Values at Risk (VaR) is \$656 509 (measured as minus the 1 % percentile of the daily P&L distribution). Assuming the returns are independent and identically distributed, we apply the square root of time rule to estimate the 1 % 10-day historical VaR as $\$656\,509 \times \sqrt{10} = \$2\,076\,065$.

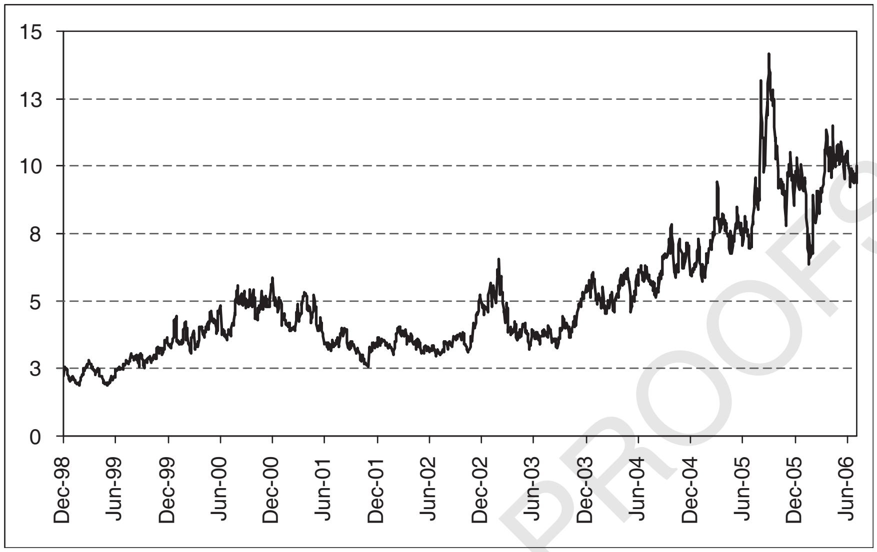

Figure 9.5 Reconstructed price series for the portfolio ($ million)

Concerned about the high probability of large losses on the portfolio, the trader decides to reduce the volatility of the portfolio returns by hedging. Our question is, which futures should he trade, and how many contracts should he buy or sell?

The principal component representation allows a close approximation of the portfolio in terms of its sensitivities to the four major risk factors. We first express the return on each constant maturity futures using its principal component representation. For instance,

$$R_{\mathrm{m1}}^{\mathrm{WTI}} \approx 0.2594P_1 + 0.1326P_2 + 0.3522P_3 + 0.0562P_4,$$

where $R_{\mathrm{m1}}^{\mathrm{WTI}}$ is the daily return on the 1-month crude oil futures and P_1, P_2, P_3 and P_4 are the first four principal components. Then we map the portfolio return to the principal component risk factors using the portfolio representation shown in Table 9.5. We obtain:

$$\text{P\&L} = 2.7914P_1 - 4.2858P_2 - 17.0186P_3 - 5.56101P_4$$

where the coefficients are measured in millions of US dollars.

This representation tells that if the first principal component shifts up by 1 % leaving the other components fixed then the portfolio will gain about 1 % of $2.791 million, i.e. about $27 910. The first component is where all futures of the same maturities move by approximately the same amount.[2] The largest sensitivity of the portfolio is to the third principal component. Thus a rise in price on the short term heating oil contract is the largest risk exposure of this portfolio. It is not easy to see this from the portfolio composition. Nevertheless, we shall now confirm this by showing that selling the 1-month heating oil

[2] From Table 9.5 we know that a 1 % upward shift in the first component implies that the 1-month crude oil futures price increases by 0.2594 %, the 1-month heating oil futures price increases by 0.275 %, the 1-month gasoline futures price increases by 0.2835 %; and so on for the 2-month futures.

contract is indeed the best hedge for the portfolio amongst all single contract futures hedges, according to the reduction in P&L volatility that it achieves.

We now consider a partial hedge using a single futures contract and targeting a reduction on P&L volatility from over \$3.5 million to less than \$2.5 million per annum. For each of the futures contracts in turn, we calculate how many futures contracts of this type we need to sell to minimize the variance of the hedge portfolio P&L.

The results are summarized in Table 9.6. For each commodity we first report the risk factor P&L volatility, and the correlation between this risk factor P&L and the portfolio P&L. Below this we show the number of contracts on the futures that should be sold to minimize the variance of the hedged portfolio's P&L. The futures having the highest correlation with the reconstructed portfolio P&L are the 1-month heating oil futures (P&L correlation = 0.746) and the corresponding minimum variance hedge ratio implies that 145.45 contracts on the 1-month heating oil futures should be sold, and this trade effects the largest possible reduction in volatility compared with any other trade on a single futures contract.

Table 9.6 Minimum variance hedges to reduce to volatility of the futures portfolio

Risk factor	**WTI m1**	**WTI m2**	**WTI m3**	**WTI m4**	**WTI m5**	**WTI m6**
Risk factor P&L Volatility (\$)	13.23	12.26	11.52	10.97	10.55	10.20
Correlation	0.411	0.441	0.440	0.440	0.440	0.441
No. contracts	111.44	128.88	137.10	143.72	149.56	155.17
Hedged portfolio P & L volatility (\$)	\$3 269 509	\$3 219 570	\$3 219 902	\$3 221 064	\$3 221 055	\$3 218 688
1% 10 day historical VaR (\$)	\$1 939 370	\$1 948 219	\$1 976 805	\$1 976 222	\$1 974 615	\$1 975 345
Risk factor	**HO m1**	**HO m2**	**HO m3**	**HO m4**	**HO m5**	**HO m6**
Risk factor P&L volatility (\$)	18.39	16.79	15.70	14.86	14.22	13.67
Correlation	0.746	0.739	0.722	0.699	0.682	0.673
No. contracts	145.45	157.80	164.84	168.64	172.02	176.52
Hedged portfolio P & L volatility (\$)	\$2 389 086	\$2 416 350	\$2 483 014	\$2 565 397	\$2 622 976	\$2 653 691
1% 10 day historical VaR (\$)	\$1 472 834	\$1 509 869	\$1 588 319	\$1 725 051	\$1 791 145	\$1 852 938
Risk factor	**UL m1**	**UL m2**	**UL m3**	**UL m4**	**UL m5**	**UL m6**
Risk factor P&L volatility (\$)	22.20	18.21	15.98	14.81	14.09	13.71
Correlation	0.661	0.679	0.703	0.740	0.682	0.682
No. contracts	106.82	133.81	157.67	179.24	173.56	178.55
Hedged portfolio P&L volatility (\$)	\$2 690 058	\$2 631 691	\$2 552 091	\$2 411 291	\$2 622 860	\$2 621 616
1% 10 day historical VaR (\$)	\$1 501 297	\$1 479 477	\$1 458 825	\$1 366 632	\$1 642 830	\$1 599 100

The hedge is effected using exchange traded heating oil futures that are equivalent, under the mapping described in Section 9.1, to 145.45 contracts on the 1-month heating oil futures. The result will be a portfolio with a historical P&L volatility of \$2.389 million, compared with over \$3.5 million without this hedge. Similarly, the hedge reduces the 1 % 10-day historical VaR from \$2.076 million to \$1.473 million.

Other single contract hedges can also reduce the risk considerably. For instance, taking a position equivalent to selling 179.24 of the 4-month futures contract on unleaded gasoline would reduce the P&L volatility to \$2.411 million and the 1 % 10-day VaR to \$1.367

million. Clearly, hedging with more than one futures contract would effect an even greater reduction in portfolio risk. For instance, using both the 1-month heating oil and 4-month gasoline futures in a single hedge, we determine the minimum variance hedge ratios using multiple regression.[3] The ordinary least squares estimated model, with t-statistics below the coefficients in parentheses is:

$$\text{P\&L} = -2.603_{\,(-0.809)} + 83.840_{\,(18.923)}\text{P\&L}^{\text{HO}}_{\text{m1}} + 97.928_{(17.802)}\,\text{P\&L}^{\text{UL}}_{\text{m4}}$$

Taking positions equivalent to 83.84 contracts on the 1-month heating oil futures and 97.928 contracts on the 4-month unleaded gasoline futures reduces the portfolio P&L volatility to 2.217 million and the 1 % 10-day VaR to $1.274 million.

9.5 CONCLUSIONS

This chapter has used a practical and realistic example to demonstrate how to identify the key risk factors of energy futures portfolios. These are the principal components of a set of constant maturity futures on related energy products – in this case, crude oil, heating oil and unleaded gasoline. Due to the very high correlation of these futures, PCA is a statistical tool that serves to reduce the dimensions of the risk factor space considerably. In our case we reduced dimensions from 18 constant maturity futures to just four principal components. Moreover, using PCA we can identify the key risk factors of the portfolio. For instance, we showed that the most common types of movement in energy futures term structures is a similar and simultaneous shift and tilt in crude oil, heating oil and unleaded gasoline futures.

Then we considered a particular portfolio, and used PCA to demonstrate that the greatest risk exposure was to short term heating oil futures, a fact that is not evident from a direct examination of the portfolio composition. We demonstrated empirically that hedging with a position that is equivalent to 145.45 contracts on the 1-month heating oil futures reduces the portfolio P&L volatility from over $3.5 million to $2.389 million, and reduces the 1 % 10-day historical VaR from $2.076 million to $1.473 million. Composite hedging with more than one futures contract reduces the P&L volatility and VaR even further.

9.6 REFERENCES

Alexander, C. (2008) *Market Risk Analysis, Volume III: Pricing, Hedging and Trading Financial Instruments*. John Wiley and Sons Ltd, Chichster.

[3] See Alexander (2008), pages 111–113.