CHAPTER SM

Statistical Models of Operational Loss

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Au: "Basel Accord" and "statistical distributions" terms do not appear in text.

Abstract: Under the Basel II accord there are pillar 1 charges for operational risks. These may be assessed using: the basic indicator approach, where the capital charge is a multiple of gross income; the standardized approach, which is similar to the basic indicator approach except being specific to business line; or the Advanced Measurement Approach (AMA), which is the subject of this chapter. The AMA entails the implementation of a statistical model for operational risk assessment. An actuarial approach is standard, whereby statistical distributions are fit to loss frequency and loss severity, the total loss distribution being the compound of these. The models are implemented using a variety of data, including internal and external losses and subjective estimates or scenarios.

Keywords: Basel Accord, regulatory capital, economic capital, operational risks, Advanced Measurement Approach (AMA), severity, frequency, statistical distributions, Bayesian estimation, scenario analysis, simulation, aggregation, copula

The purpose of this chapter is to give a theoretical but pedagogical introduction to the advanced statistical models that are currently being developed to estimate operational risks, with many examples to illustrate their applications in the financial industry.

OPERATIONAL RISKS

Let's begin with some definitions of the *operational risks* facing financial institutions. These risks may be categorized according to the frequency of occurrence and their impact in terms of financial loss. Following this is a general discussion of the data that are necessary for measuring these risks. More detailed descriptions of loss history and/or key risk indicator (KRI) data are given in later sections. The focus of this introductory discussion is to highlight the data availability problems with the risks that will have the most impact on the capital charge—the low-frequency, high-impact risks. Internal data on such risks are, by definition, sparse, and will need to be augmented by "soft" data, such as that from scorecards, expert opinions, or from an external data consortium. All these soft data have a subjective element and should therefore be distinguished from the more objective, or "hard" data that is obtained directly from the historical loss experiences of the bank. In the next section we will introduce Bayesian estimation, which is one of the methods that can be employed to combine data from different sources to obtain parameter estimates for the loss distribution.

Definitions of Operational Risks

After much discussion between regulators and the industry, operational risk has been defined by the Basel Committee as "the risk of financial loss resulting from inadequate or failed internal processes, people and systems or from external events." It includes legal risk but not reputational risk (where decline in the firm's value is linked to a damaged reputation) or strategic risk (where, for example, a loss results from a misguided business decision). The Basel Committee (2001b) working paper also defines seven distinct types of operational risk:

1. Internal Fraud
2. External Fraud
3. Employment Practices and Workplace Safety
4. Clients, Products, and Business Practices
5. Damage to Physical Assets
6. Business Disruption and System Failures
7. Execution, Delivery, and Process Management

Detailed definitions of each risk type are given in Annex 2 of the Basel working paper.

Historical operational loss experience data has been collected in data consortia such as Op-Vantage and ORX. The latter is a not-for-profit data consortium that is incorporated in Basel as a Swiss association of major banks. Data collection started in January 2002, building on the expertise of existing commercial data consortia. According to data from the Op-Vantage web site, the total losses recorded over a period of more than 10 years on more than 7,000 loss events greater than US$1 million was about US$272 billion of losses. More than 70% of the total losses recorded were due to the risk type "Clients, Products, and Business Practices." These are the losses arising from unintentional or negligent failure to meet a professional obligation to specific clients, or from the nature or design of a product. They include the fines and legal losses arising from breech of privacy, aggressive sales, lender liability, improper market practices, money laundering, market manipulation, insider trading, product flaws, exceeding client exposure limits, disputes over performance of advisory activities, and so forth. The other two significant loss categories are Internal Fraud and External Fraud, both relatively low-frequency risks for investment banks: Normally, it is only in the retail banking sector that external fraud (e.g., credit card fraud) occurs with high frequency.

Frequency and Severity

The seven types of operational risk may be categorized in terms of *frequency* (the number of loss events during a certain time period) and *severity* (the impact of the event in terms of financial loss). Table SM.1, which is based on the results from the Basel Committee (2002), indicates the typical frequency and severity of each risk type that may arise for a typical bank with investment, commercial, and retail operations.

Banks that intend to use the *Advanced Measurement Approach (AMA)* proposed by the Basel Committee (2001b) to quantify the operational risk capital requirement (ORR) will be required to measure the ORR for each risk type in each of the following eight lines of business:

1. Investment Banking (Corporate Finance)
2. Investment Banking (Trading and Sales)
3. Retail Banking
4. Commercial Banking
5. Payment and Settlement
6. Agency Services and Custody
7. Asset Management
8. Retail Brokerage

Depending on the bank's operations, up to 56 separate ORR estimates will be aggregated over the matrix shown in Table SM.2 to obtain a total ORR for the bank.

In each cell of Table CM.2, the upper (shaded) region indicates the frequency of the risk as high (H), medium (M), or low (L), and the lower region shows the severity also as high, medium, or low. The indication of typical frequency and severity given in this table is very general and would not always apply. For example, Employment Practices and Workplace Safety, Damage to Physical Assets, and Business Disruptions and System Failure are all classified in

Table SM.1 Frequency and Severity of Operational Risk Types

Risk	Frequency	Severity
Internal Fraud	Low	High
External Fraud	High/Medium	Low/Medium
Employment Practices and Workplace Safety	Low	Low
Clients, Products, and Business Practices	Low/Medium	High/Medium
Damage to Physical Assets	Low	Low
Business Disruption and System Failures	Low	Low
Execution, Delivery, and Process Management	High	Low

Table SM.2 Frequency and Severity by Business Line and Risk Type

	Internal Fraud	External Fraud	Employment Practices and Workplace Safety	Clients, Products, and Business Practices	Damage to Physical Assets	Business Disruption and System Failures	Execution, Delivery, and Process Management
Corporate Finance	L[1]	L	L	L[1]	L	L	L
	H[1]	M	L	H[1]	L	L	L
Trading and Sales	L[1]	L	L	M	L	L	H[2]
	H[1]	L	L	M	L	L	L
Retail Banking	L	H[2]	L	M	M	L	H[2]
	M	L[2]	L	M	L	L	L
Commercial Banking	L[1]	M	L	M	L	L	M
	H[1]	M	L	M	L	L	L
Payment and Settlement	L	L	L	L	L	L	H[2]
	M	L	L	L	L	L	L[2]
Agency and Custody	L	L	L	L	L	L	M
	M	L	L	M	L	L	L
Asset Management	L[1]	L	L	L[1]	L	L	M
	H[1]	L	L	H[1]	L	L	L
Retail Brokerage	L	L	L	L	L	M	M
	M	M	L	M	L	L	L

Notes: Upper (shaded) region indicates the frequency of the risk as high (H), medium (M) or low (L), and the lower region shows the severity also as high, medium or low.
[1]Indicates the low-frequency, high-severity risks that could jeopardize the whole future of the firm.
[2]Indicates the high-frequency, low-severity risks that will have high expected loss but relatively low unexpected loss.
[3]Indicates the operational risk types that are likely to have high unexpected losses.

the table as low/medium frequency, low severity—but this would not be appropriate if, for example, a bank has operations in a geographically sensitive location.

In certain cells, the number 1, 2, or 3 is shown. The number 1 indicates the low-frequency, high-severity risks that could jeopardize the whole future of the firm. These are the risks associated with loss events that will lie in the very upper tail of the total annual loss distribution for the bank. Depending on the bank's direct experience and how these risks are quantified, they may have a huge influence on the total ORR of the bank. Therefore, new insurance products, which cover such events as internal fraud, or securitization of these risks with OpRisk "catastrophe" bonds are some of the mitigation methods that should be considered by the industry.

The cells where the number 2 is shown indicate the high-frequency, low-severity risks that will have high expected loss but relatively low unexpected loss. These risks, which include credit card fraud and some human risks, should already be covered by the general provisions of the business. Assuming unexpected loss is quantified in the proper way, they will have little influence on the ORR. Instead, these are the risks that should be the focus of improving process management to add value to the firm. The operational risk types that are likely to have high unexpected losses are indicated by the number 3 in Table SM.2. These risks will have a substantial impact on the ORR. These medium-frequency, medium-severity risks should therefore be a main focus of the quantitative approaches for measuring operational risk capital.

Probability-Impact Diagrams

In the quantitative analysis of operational risks, frequency and severity are regarded as random variables. Expected frequency may be expressed as Np, where N is the number of events susceptible to operational losses, and p is the probability of a loss event. Often, a proxy for the number of events is a simple volume indicator such as gross income, and/or it could be the focus of management targets for the next year. In this case, it is the loss probability rather than loss frequency that will be the focus of operational risk measurement and management, for example, in Bayesian estimation and in the collection of scorecard data.

A probability-impact diagram, or "risk map," such as that shown in Figure SM.1, is a plot of expected loss frequency versus expected severity (impact) for each risk type/line of business. Often, the variables are plotted on a logarithmic scale because of the diversity of frequency and impacts of different types of risk. This type of diagram is a useful visual aid to identifying which risks should be the main focus of management control, the intention being to reduce either frequency or impact (or both) so that the risk lies within an acceptable region.

In Figure SM.1 the risks that give rise to the black crosses in the dark-shaded region should be the main focus of management control; the reduction of probability and/or impact, indicated by the arrows in the diagram, may bring these into the acceptable region (with the white background) or the warning region (the light-shaded region).

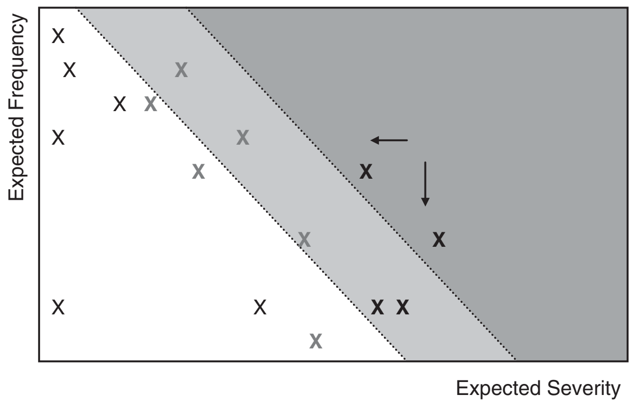

Figure SM.1: A Probability-Impact Diagram

Data Considerations

The Basel Committee (2001b) states that banks that wish to quantify their *regulatory capital* (ORR) using a loss distribution model will need to use historical data based on actual loss experience, covering a period of at least three years (preferably five years), that are relevant to each risk type and line of business. But data on the frequency and severity of historical losses are difficult to obtain. Internal historical data on high-frequency risks such as Execution, Delivery, and Process Management should be relatively easy to obtain, but since these risks are also normally of low impact, they are not the important ones from the point of view of the ORR. The medium-frequency, medium-impact risks, such as Clients, Products, Business Practices, and the low-frequency, high-impact risks, such as Internal Fraud, are the most important risks to measure from the regulatory capital perspective. Thus, the important risks are those that, by definition, have little internal data on historical loss experience.

With little internal data, the estimates of loss frequency and severity distribution parameters will have large sampling errors if they are based only on these. *Economic capital* forecasts will therefore vary considerably over time, and risk budgeting will be very difficult. Consequently, the bank will need to consider supplementing its internal data with data from other sources. These could be internal scorecard data based on expert opinion or data from an external consortium.

Scorecards

Even when loss event data are available, they are not necessarily as good an indication of future loss experience as scorecard data. However, scorecard data are very subjective:

- As yet we have not developed the industry standards for the KRIs that should be used for each risk type. Thus, the choice of risk indicators themselves is subjective.
- Given a set of risk indicators, probability and impact scores are usually assigned by the "owner" of the operational risk. Careful design of the management process (e.g., a "no blame" culture) is necessary to avoid subjective bias at this stage.
- Not only are the scores themselves subjective, but when scorecard data are used in a loss distribution model, the scores need to be mapped, in a more or less subjective manner, to monetary loss amounts. This is not an easy task, particularly for risks that are associated with inadequate or failed people or management processes—these are commonly termed "human risks."

To use scorecard data in the AMA, the minimum requirement is to assess both expected frequency and expected severity quantitatively, from scores that may be purely qualitative (see Table SM.3). For example, the score "very unlikely" for a loss event, might first translate into a probability, depending on the scorecard design. In that case, the expected frequency must be quantified by assuming a fixed number N of events that are susceptible to operational loss. In the scorecard below, $N = 10$ events per month. The scorecard will typically specify a range of expected frequency, and the exact point in this range should be fixed by *scenario analysis* using comparison with loss experience data. If internal data are not available, then external data should be used to validate the scorecard.

The basic Internal Measurement Approach (IMA) requires only expected frequency and expected severity, but for the general IMA formula given later in this chapter, and the simulation of the total loss distribution explained later in this chapter, higher moments of the frequency and severity distributions must also be recovered from the scorecard. Uncertainty scores are also needed; that is, the scorer who forecasts an expected loss severity of £40,000 must also answer the question, "How certain are you of this forecast?" Although the loss severity standard deviation will be needed in the AMA model, it would be misleading to give a score in these terms. This is because standard deviations are not invariant under monotonic transformations. The standard deviation of logarithm (log) severity may be only half as large as the mean log severity at the same time as the standard deviation of severity is twice as large as the mean severity. So if standard deviation were used to measure uncertainty, we would conclude from this severity data that we are "fairly uncertain," but the conclusion from the same data in log form would be that we are "certain." However, percentiles are invariant under monotonic transformations,

Table SM.3 Scorecard Data

Definition	**Probability, p**	**Expected Frequency, Np**
Almost impossible	[0, 0.01]%	Less than once in 10 years
Rare	[0.1, 1]%	Between 1 per year and 1 per 10 years
Very unlikely	[1, 10]%	Between 1 per month and 1 per year
Unlikely	[10, 50]%	Between 1 and 5 per month
Likely	[50, 90]%	Between 5 and 9 per month
Very likely	[90, 100]%	More than 9 per month

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Table SM.4

Definition	Upper 99 Percentile—mean (as a multiple of the mean)
Extremely uncertain	5 or more
Very uncertain	2–5
Fairly uncertain	1–2
Fairly certain	0.5–1
Very certain	0.1–0.5
Extremely certain	Up to 0.1

so uncertainty scores should be expressed as upper percentiles, that is, as "worst case" frequencies and severities, for example as in Table SM.4.

Despite the subjectivity of scorecard data, there are many advantages in their use, not the least of which is that scores can be forward looking. Thus, they may give a more appropriate view of the future risk than measures that are based purely on historical loss experience. Moreover, there are well-established quantitative methods that can account for the subjectivity of scorecard data in the proper manner. These are the Bayesian methods that will be introduced in the next section.

External Data

The Basel Committee (2001b) states: "The sharing of loss data, based on consistent definitions and metrics, is necessary to arrive at a comprehensive assessment of operational risk. For certain event types, banks may need to supplement their internal loss data with external, industry loss data." However, there are problems when sharing data within a consortium. Suppose a bank joins a data consortium and that Delboy Financial Products Bank (DFPB) is also a member of that consortium. Also suppose that DFPB has just reported a very large operational loss—say a rogue trader falsified accounts and incurred losses in the region of US$1 billion. If a bank were to use that consortium data as if it were internal data, only scaling the unexpected loss by taking into account its capitalization relative to the total capitalization of the banks in the consortium, the estimated ORR will be rather high, to say the least.

For this reason, the Basel Committee working paper also states:

> *The bank must establish procedures for the use of external data as a supplement to its internal loss data . . . they must specify procedures and methodologies for the scaling of external loss data or internal loss data from other sources. New methods for combining internal and external data are now being developed.*

Also, statistical methods that have been established for centuries are now being adapted to the operational loss distribution framework, and these are described in the next section.

BAYESIAN ESTIMATION

Bayesian estimation is a parameter estimation method that combines "hard" data that is thought to be more objective, with "soft" data that can be purely subjective. In operational risk terms, the "hard" data may be the recent and relevant internal data and the "soft" data could be from an external consortium, or purely subjective data in the form of risk scores based on opinions from industry experts or the owner of the risk. Soft data could also be past internal data that, following a merger, acquisition, or sale of assets, are not so relevant today. (When a bank's operations undergo a significant change in size, such as would be expected following a merger or acquisition or a sale of assets, it may not be sufficient to simply rescale the capital charge by the size of its current operations. The internal systems, processes, and people are likely to have changed considerably, and in this case the historic loss event data would no longer have the same relevance today.)

Bayesian estimation methods are based on two sources of information—the soft data are used to estimate a "prior density" for the parameter of interest, and the hard data are used to estimate another density for the parameter that is called the "sample likelihood." These two densities are then multiplied to give the "posterior density" on the model parameter.

Figure SM.2 illustrates the effect of different priors on the posterior density. The hard data represented by

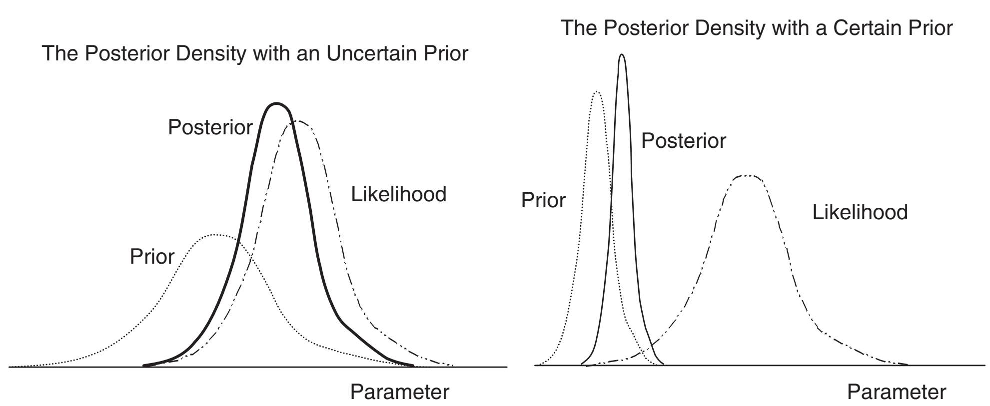

Figure SM.2: Prior, Likelihood, and Posterior Densities the likelihood is the same in both cases, but the left-hand figure illustrates the case when soft data are uncertain, and the second figure illustrates the case that soft data are certain. (Uncertain [i.e., vague] priors arise, for example, when the data in the external data consortium [for this risk type and line of business] are either sparse or very diverse, or when the industry expert or risk owner is uncertain about the scores recorded. Certain [i.e., precise] priors arise, for example, when there are plenty of homogeneous data in the consortium or when the industry expert or the owner of the risk is fairly certain about their scores.)

If desired, a point estimate of the parameter may be obtained from the posterior density, and this is called the Bayesian estimate. The point estimate will be the mean, or the mode or the median of the posterior density, depending on the loss function of the decision maker. (Bayesians view the process of parameter estimation as a *decision* rather than as a statistical objective. That is, parameters are chosen to minimize expected loss, where expected loss is defined by the posterior density and a chosen loss function. Classical statistical estimation, however, defines a statistical objective such as “sum of squared errors” or “sample likelihood” which is minimized [or maximized] and thus the classical estimator is defined.)

In this section we will assume that the decision maker has a quadratic loss function, so that the Bayesian estimate of the parameter will be the mean of the posterior density.

We say that the prior is “conjugate” with the likelihood if it has the same parametric form as the likelihood and their product (the posterior) is also of this form. For example, if both prior and likelihood are normal, the posterior will also be normal. Also, if both prior and likelihood are beta densities, the posterior will also be a beta density. The concept of conjugate priors allows one to combine external and internal data in a tractable manner. With conjugate priors, posterior densities are easy to compute analytically; otherwise, one could use *simulation* to estimate the posterior density. We now illustrate the Bayesian method with examples on the estimation of loss frequency and severity distribution parameters using both internal and external data.

Bayesian Estimation of Loss Severity Parameters

It is often the case that uncertainty in the internal sample is less than the uncertainty in the external sample, because of the heterogeneity of members in a data consortium. Thus, Bayesian estimates of the expected loss severity will often be nearer the internal mean than the external mean, as illustrated in the next section. Note the importance of this for the bank that joins the consortium with Delboy Financial Products Bank: DFPB made a huge operational loss last year; therefore, if the bank were to use classical estimation methods (such as maximum likelihood) to estimate μ_L as the average loss in the combined sample, this would be very large indeed. However, the opposite would apply if the bank were to use Bayesian estimation! Here, the effect of DFPB’s excessive loss will be to increase the standard deviation in the external sample considerably, and this increased uncertainty will affect the Bayesian estimate so that it will be closer to the internal sample mean than mean in the data consortium.

Another interesting consequence of the Bayesian approach to estimating loss severity distribution parameters when the parameters are normally distributed is that the Bayesian estimate of the standard deviation of the loss severity will be less than both the internal estimate and the external estimate of standard deviation. In the illustration below, the Bayesian estimate of the standard deviation was \$0.83 million, which is less than both the internal estimate (\$1 million) and the external estimate (\$1.5 million). This reduction in overall variance reflects the value of more information: In simple terms, by adding new information to the internal (or external) density, the uncertainty must be decreased.

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Note the importance of this statement for the bank that measures the ORR using an advanced approach. By augmenting the sample with external data, the standard deviation in loss severity will be reduced, and this will tend to decrease the estimate of the ORR. However, the net effect on the ORR is indeterminate for two reasons: (1) the combined sample estimate of the mean loss severity may be increased—and this will tend to increase the ORR; and (2) the ORR also depends on the combined estimate for the parameters of the loss frequency distribution.

Estimating the Mean and Standard Deviation of a Loss Severity Distribution

Suppose that the internal and external data on losses (over a threshold of \$1 million) due to a given type of operational risk are shown in Table SM.5. Based on the internal data only, the mean and standard deviation of loss severity are \$2 million and \$1 million, respectively; based on the external data only, the mean and standard deviation of loss severity are \$3 million and \$1.5 million, respectively. Note that the uncertainty, as measured by the standard deviation, is larger in the external data and this is probably due to the heterogeneity of banks in the consortium.

Table SM.5 Internal and External Loss Data

	Internal	External
	1.25	3.2
	1.35	1.15
	2.75	6.35
	1.15	1.45
	3.65	4.5
	1.85	2.75
		1.8
		2.3
		3.65
		4.25
		1.6
		4.9
		2.3
		3.2
		1.85
Mean	2.00	3.00
Std. Dev.	1.00	1.50

We now show that the Bayesian estimate of μ_L, based on both sources of data, will be closer to the estimate of μ_L that is based only on internal data. The intuition for this is that there is more uncertainty in the external data, so the posterior density will be closer to the density based on the internal data (this is the situation shown in the left-hand side of Figure SM.2), and the Bayesian estimate is the mean of the posterior density.

In Bayesian estimation the parameters are regarded as random variables. Assume that the prior density and the sample likelihood are normal distributions on μ_L (as would be the case if, for example, the loss severity distribution is normal). Therefore, the posterior density, being the product of these, will also be normal. From this, it follows that the Bayesian estimate of the mean loss severity, which combines both internal and external data, will be a weighted average of the external sample mean and the internal sample mean, where the weights will be the reciprocals of the variances of the respective distributions. In the example data shown in the table above, the Bayesian estimate for the expected loss severity is therefore:

$$[(2/1^2) + (3/(1.5)^2)]/[(1/1^2) + (1/(1.5)^2)] = \$2.3\text{ million}$$

It is nearer the internal sample mean (\$2 million) than the external sample mean (\$3 million) because the internal data has less variability than the external data. Similarly, the Bayesian estimate of the loss severity standard deviation will be:

$$\{[(1/(1^2) + 1/(1.5^2)]^{-1}\}^{0.5} = \$0.83\text{ million}$$

It is less than both the internal and the external standard deviation estimates because of the additional value of information.

Note that the maximum likelihood estimates that are based on the combined sample with no differentiation of data source are \$2.7 million for the mean and \$1.43 for the standard deviation.

This example will be continued in Figure SM.4, where it will be shown that the estimated capital charges will be significantly different, depending on whether parameter estimates are based on Bayesian or classical estimation.

Bayesian Estimation of Loss Probability

Now let us consider how Bayesian estimation may be used to combine hard data and soft data on loss probability. As noted at the beginning of this chapter, an important parameter of the loss frequency distribution is the mean number of loss events over the time period. This is the expected frequency, and it may be written as Np, where N is the total number of events that are susceptible to operational losses and p is the probability of a loss event. It is not always possible to estimate N and p separately and, if only a single data source is used, it is not necessary (see the next section).

However, regulatory capital charges are supposed to be forward looking, so the value for N used to calculate the ORR should represent a forecast over the time period (one year is recommended in Basel Committee, 2001). Thus, we should use a target or projected value for N—assuming this can be defined by the management—and this target could be quite different from its historical value. But can N be properly defined—and even if it can be defined, can it be forecast? The answer is yes, but only for some risk types and lines of business. For example, in Clients, Products, and Business Practices" or in Internal Fraud in the line of business Trading and Sales, the value for N should correspond to the target number of ongoing deals during the forthcoming year, and p should correspond to the probability of an ongoing deal incurring an operational loss of this type. Assuming one can define a target value for N, the expected frequency will then be determined by the estimate of p, the probability of an operational loss.

Bayesian estimates for probabilities are usually based on beta densities, which take the form:

$$f(p) \propto p^a(1-p)^b \quad 0 < p < 1 \qquad \text{(SM.1)}$$

We use the notation $\propto$ to express the fact that (SM.1) is not a proper density—the integral under that curve is not equal to one because the normalizing constant, which involves the gamma function, has been omitted. However, normalizing constants are not important to carry through at every stage: If both prior and likelihood are beta densities, the posterior will also be a beta density, and we can normalize this at the end. It is easy to show that a beta density given by (SM.1) has

$$\text{Mean} = (a+1)/(a+b+2)$$
$$\text{Variance} = (a+1)(b+1)/(a+b+2)^2(a+b+1)$$

The mean will be the Bayesian estimate for the loss probability p corresponding to the quadratic loss function, where a and b are the parameters of the posterior density. The following examples use the formula for the variance to obtain the parameters a and b of a prior beta density based on subjective scores of a loss probability.

Estimating the Loss Probability Using Internal Data Combined with (a) External Data and (b) Scorecard Data

Here are two examples that show how to calculate a Bayesian estimate of loss probability using two sources of data. In each case, the hard data will be the internal data given in the previous section, assuming these data represented a total of 60 deals. Thus, with six loss events, the internal loss probability estimate was $6/60 = 0.1$. This is, in fact, the maximum likelihood estimate corresponding to the sample likelihood, which is a beta density:

$$f_1(p) \propto p^6(1-p)^{54}$$

Now consider two possible sources of soft data: (1) the external data in the table in the previous section, which we now suppose represented a total of 300 deals; and (2) scorecard data that has assigned an expected loss probability of 0.05 with an uncertainty surrounding this score of 20%. That is, the ± 1 standard error bounds for the score of 0.05 are $0.05 \pm (0.0570.2) = [0.04, 0.06]$ and the ± 3 standard error bounds for the score of 0.05 are $0.05 \pm (0.0570.6) = [0.02, 0.08]$.

In case (1) the external loss probability estimate is 15/300 = 0.05 and the prior is the beta density

$$f_2(p) \propto p^{15}(1-p)^{285}$$

The posterior density representing the combined data, which is the product of this prior with the likelihood beta density $f_1(p)$, is another beta density, namely:

$$f_3(\mathrm{p}) \propto p^{21}(1-p)^{339}$$

The mean of this density gives the Bayesian estimate of p as $\hat{\mathrm{p}} = 22/362 = 0.06$. Note that the classical maximum likelihood estimate that treats all data as the same is $21/360 = 0.058$.

In case (2) a prior beta density that has mean 0.05 and standard deviation 0.2 is

$$f_2(p) \propto p^{5}(1-p)^{113}$$

and this can be verified using the mean and variance formulae for a beta density above. The posterior becomes

$$f_3(p) \propto p^{11}(1-p)^{167}$$

The mean of this density gives the Bayesian estimate of p as $\hat{\mathrm{p}} = 12/180 = 0.0667$.

The bank should then use its target value for N to compute the expected number of loss events over the next year as $N\,\hat{\mathrm{p}}$. We will return to the examples in Figures SM.3 and SM.4 later in this chapter when the operational risk capital requirement calculations based on different type of parameter estimates will be compared, using targets for N and classical and Bayesian estimates for p, μ_L, and σ_L.

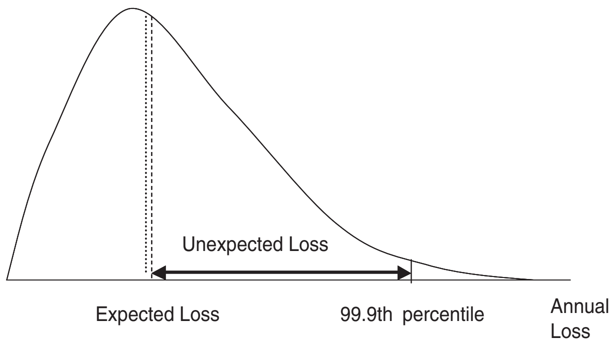

Figure SM.3: Unexpected Loss

INTRODUCING THE ADVANCED MEASUREMENT APPROACHES

At first sight, a number of advanced measurement approaches to estimating operational risk capital requirements appear to be proposed in the Basel Committee (2001b) working paper "CP2.5." A common phrase used by regulators and supervisors has been "Let a thousand flowers bloom." However, in this section and the next we show that the Internal Measurement Approach (IMA) of CP2.5 just gives an analytic approximation for the unexpected loss in a typical actuarial loss model. The only difference between the IMA and the Loss Distribution Approach (LDA) is that the latter uses simulation to estimate the whole loss distribution, whereas the former merely gives an analytic approximation to the unexpected loss. To be more precise, if uncertainty in loss severity is modeled by a standard deviation, but no functional form is imposed on the severity distribution, there is a simple formula for the unexpected annual loss, and that is the IMA formula. Also, the scorecard approach that was proposed in the Basel working paper is referring to the data, not the statistical methodology. In fact, there is only one advanced measurement approach, and that is the actuarial approach.

A General Framework for the Advanced Measurement Approach

The operational risk capital requirement based on the AMA will, under the current proposals, be the unexpected

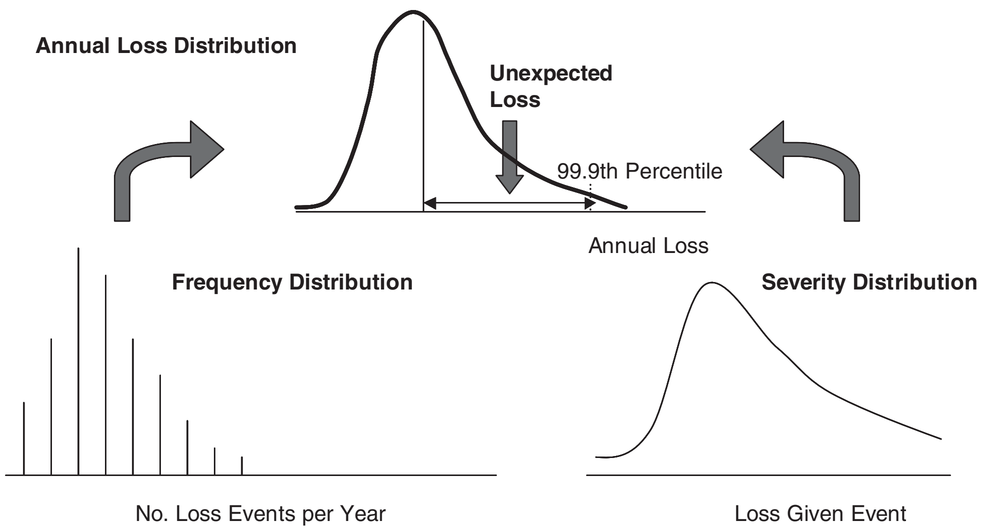

Figure SM.4: Compounding Frequency and Severity Distributions loss in the total loss distribution corresponding to a confidence level of 99.9% and a risk horizon of one year. This unexpected loss is illustrated in Figure SM.3: It is the difference between the 99.9th percentile and the expected loss in the total operational loss distribution for the bank. Losses below the expected loss should be covered by general provisions, and losses above the 99.9th percentile could bankrupt the firm, so they will need to be controlled. Capital charges are to cover losses in between these two limits: The common but rather unfortunate term for this is "unexpected loss."

Figure SM.4 shows how the annual loss distribution is a compound distribution of the loss frequency distribution and the loss severity distribution. That is, for a given operational risk type in a given line of business, we construct a discrete probability density $h(\boldsymbol{n})$ of the number of loss events n during one year, and continuous conditional probability densities $g(x|n)$ of the loss severities, x, given there are n loss events during the year. The annual loss then has the compound density:

$$f(x) = \sum_{n=0}^{\infty} h(n) g(x|n) \qquad \text{(SM.2)}$$

Following the current Basel II proposals, the bank may consider constructing an annual loss distribution for each line of business and risk type. It is free to use different functional forms for the frequency and severity distributions for each risk type/line of business. The aggregation of these loss distributions into a total annual operational loss distribution for the bank will be discussed later in this chapter.

Functional Forms for Loss Frequency and Severity Distributions

Consider first the frequency distribution. At the most basic level we can model this by the binomial distribution $B(N, p)$ where N is the total number of events that are susceptible to an operational loss during one year, and p is the probability of a loss event. Assuming independence of events, the density function for the frequency distribution is then given by

$$h(n) = \binom{N}{n} p^n (1-p)^{N-n} \quad n = 0, 1, \ldots, N \qquad \text{(SM.3)}$$

The disadvantage with the binomial density (SM.3) is that one needs to specify the total number of events, N. However, when p is small the binomial distribution is well approximated by the Poisson distribution, which has a single parameter λ, corresponding to the expected frequency of loss events – that is Np in the binomial model. Thus low frequency operational risks may have frequency densities that are well captured by the Poisson distribution, with density function

$$h(n) = \frac{\lambda^n \exp(-\lambda)}{n!} \quad n = 0, 1, 2 \ldots. \qquad \text{(SM.4)}$$

Otherwise a better representation of the loss frequency may be obtained with a more flexible functional form, a two-parameter distribution such as the negative binomial distribution with density function

$$h(n) = \binom{\alpha + n - 1}{n} \left(\frac{1}{1+\beta}\right)^{\alpha} \left(\frac{\beta}{1+\beta}\right)^{n}$$
$$n = 0, 1, 2, \ldots.. \qquad \text{(SM.5)}$$

Turning now to the loss severity, one does not necessarily wish to choose a functional form for its distribution. In fact when one is content to model uncertainty in the loss severity directly, simply by the loss severity variance, the unexpected annual loss may be approximated by an analytic formula. The precise formula will depend on the functional form for the frequency density, and we shall examine this later in this chapter.

When setting a functional form for the loss severity distribution, a common simplifying assumption is that loss frequency and loss severity are independent. In this case, only one (unconditional) severity distribution $g(x)$ is specified for each risk type and line of business; indeed, $g(x \mid n)$ may be obtained using convolution integrals of $g(x)$.

It clearly not appropriate to assume that aggregate frequency and severity distributions are independent—for example, high-frequency risks tend to have a lower impact than many low-frequency risks. However, within a given risk type and line of business an assumption of independence is not necessarily inappropriate. Clearly, the range for severity will be not be the same for all risk types (it can be higher for low-frequency risks than for high-frequency risks) and also the functional form chosen for the severity distribution may be different across different risk types.

High-frequency risks can have severity distributions that are relatively lognormal, so that

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right) \quad (x > 0) \qquad \text{(SM.6)}$$

However, some severity distributions may have substantial leptokurtosis and skewness. In that case, a better fit is provided by a two-parameter density. Often, we use the gamma density:

$$g(x) = \frac{x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)}{\beta^{\alpha}\Gamma(\alpha)} \quad (x > 0) \qquad \text{(SM.7)}$$

where $\Gamma(.)$ denotes the gamma function or the two-parameter hyperbolic density:

$$g(x) = \frac{\exp\left(-\alpha\sqrt{\beta^2 + x^2}\right)}{2\beta B(\alpha\beta)} \quad (x > 0) \qquad \text{(SM.8)}$$

where $B(.)$ denotes the Bessell function.

Further discussion about the properties of these frequency and severity distributions will be given later in this chapter, when we will apply them to estimating the unexpected annual loss.

Comments on Parameter Estimation

Having chosen the functional forms for the loss frequency and severity densities to represent each cell in the risk type/line-of-business categorization, the user needs to specify the parameter values for all of these. The parameter values used must represent forecasts for the loss frequency and severity distributions, over the risk horizon on the model. If historical data on loss experiences are available, these may provide some indication of the appropriate parameter values. One needs to differentiate between sources of historical data, and if more than one data source is used, or in any case where data have a highly subjective element, a Bayesian approach to parameter estimation should be utilized, as explained previously in this chapter. For example, when combining internal with external data, more weight should be placed on the data with less sampling variation—often the internal data, given that external data consortia may have quite heterogeneous members.

However, the past is not an accurate reflection of the future: not just for market prices, but also for all types of risk, including operational risks. Therefore, parameter estimates that are based on historical loss experience data or retrospective operational risk scores can be very misleading. A great advantage of using scorecards and expert opinions, rather than historical loss experience, is that the parameters derived from these can be truly forward looking. Although more subjective—indeed, they may not even be linked to a historical loss experience—scorecard data may be more appropriate than historical loss event data for predicting the future risk.

The data for operational risk models are incomplete, unreliable, and/or have a high subjective element. Thus, it is clear that the parameters of the annual loss distribution cannot be estimated very precisely. Consequently, it is not very meaningful to propose the estimation of risk at the 99.9th percentile (see the comment below). Even at the 99.9th percentile, large changes in the unexpected loss arise from very small changes in parameter estimates. Therefore, regulators should ask themselves very seriously whether it is, in fact, sensible to base ORR calculations on this method.

For internal purposes, a parameterization of the loss severity and frequency distributions are useful for the scenario analysis for operational risks. For example, the management may ask questions along the following lines:

- What is the effect on the annual loss distribution when the loss probability decreases by this amount?
- If loss severity uncertainty increases, what is the effect on the unexpected annual loss?

To answer such quantitative questions, one must first specify a functional form for the loss severity and frequency densities, and then perturb their parameters.

Comments on the 99.9th Percentile

Very small changes in the values of the parameters of the annual loss distribution will lead to very large changes in the 99.9th percentile. For example, consider the three annual loss distributions shown in Figure SM.5. For the purposes of illustration we suppose that a gamma density (SM.7) is fitted to annual loss with slightly different parameters in each of the three cases. (In density 1, the parameters are $\alpha = 3$, $\beta = 6$; in density 2, they are $\alpha = 3.2$, $\beta = 5.5$; in density 3, they are $\alpha = 2.8$, $\beta = 6.5$. To the naked eye, the three distributions look the same in the upper tail, although there are slight differences around the mean.)

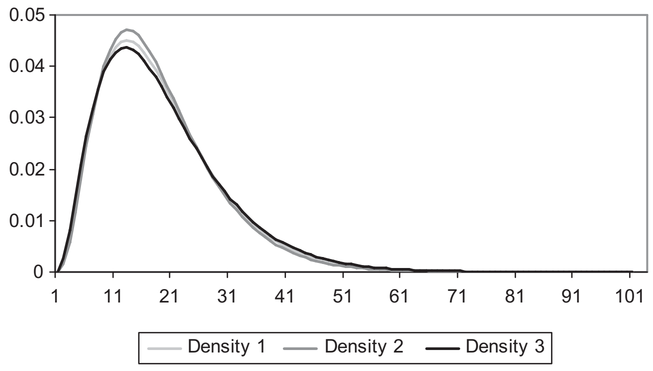

Figure SM.5: Three Similar Densities

The mean of a gamma distribution (SM.7) is $\alpha\beta$. In fact, the means shown in the first column of Table SM.6 are not very different among the three different densities. The 95th percentiles are also fairly similar, as are the unexpected losses at the 95th percentile: the largest difference (between densities 2 and 3) is $20.8 - 18.9 = 1.1$. That is, there is a 5.8% increase in the 95% unexpected loss from density 2 to density 3. However, there are very substantial differences between the 99.9th percentiles and the associated unexpected loss: Even the very small changes in fitted densities shown in Figure SM.5 can lead to a 14% increase in the ORR.

It is important to realize that the parameters of an annual operational loss distribution cannot be estimated with precision: A large quantity of objective data is necessary for this, but it is simply not there, and never will be. Operational risks will always be quantified by subjective data, or external data, whose relevance is questionable.

In the preceding example, we did not even consider the effect on the 99.9th percentile estimate from changing to a different functional form. However, the bank is faced with a plethora of possible distributions to choose from; for severity, in addition to (SM.6), (SM.7), and (SM.8), the bank could choose to use any of the extreme value distributions (as in Frachot, Georges, and Roncalli, 2001) or any mixture distribution that has suitably heavy tails. The effect of moving from one functional form to another is likely to have an even greater impact on the tail behavior than the effect of small changes in parameter estimates.

Table SM.6 Comparison of Percentiles and Unexpected Loss

		Percentile			Unexpected Loss		
	Mean	**95%**	**99%**	**99.9%**	**95%**	**99%**	**99.9%**
Density 1	18	38	50.5	67.5	20	32.5	49.5
Density 2	17.6	36.5	48.2	63.5	18.9	30.6	45.9
Density 3	18.2	39	51.5	70.5	20.8	33.3	52.3

Furthermore, later in this chapter we show that, even if there is no uncertainty surrounding the choice for individual functional forms and no uncertainty about the parameter estimates, the use of slightly different dependence assumptions will have an enormous impact on the 99.9th percentile estimate. It is clear that estimates of the 99.9th percentile of a total annual operational loss distribution will always be very, very imprecise. Nevertheless, regulators propose using the unexpected loss at the 99.9th percentile to estimate the ORR.

ANALYTIC APPROXIMATIONS TO UNEXPECTED ANNUAL LOSS

This section develops some analytic methods for estimating the regulatory capital to cover operational risks (recall that this capital is referred to as the operational risk requirement [ORR] throughout this chapter). All the analytic formulas given here are based on the Internal Measurement Approach (IMA) that has been recommended by the Basel Committee (2001b). In the course of this section, we will show how to determine the Basel "gamma" factor, thus solving a problem that has previously vexed both regulators and risk managers.

The IMA has some advantages:

- Banks and other financial institutions that implement the IMA will gain insight to the most important sources of operational risk. The IMA is not a "top-down" approach to risk capital, where capital is simply top-sliced from some gross exposure indicator at a percentage that is set by regulators to maintain the aggregate level of regulatory capital in the system. Instead, operational risk estimates are linked to different risk types and lines of business, and to the frequency and severity of operational losses. But the IMA also falls short of being a "bottom-up" approach, where unexpected losses are linked to causal factors that can be controlled by management. Having noted this, the implementation of an IMA, or indeed any loss distribution approach (LDA) is still an important step along the path to operational risk management and control (see Alexander, 200?). **Au: Pls. provide year.**
- The IMA might produce lower regulatory capital estimates than the basic indicator and standardized approaches, although this will depend very much on the risk type, the data used, and the method of estimating parameters, as we will show in two examples later in this chapter.
- The IMA gives rise to several simple analytic formulas for the ORR, all of which are derived from the basic formula given by Basel Committee (2001b). The basic Basel formula is:

$$\text{ORR} = \text{gamma} \times \text{expected annual loss} = \gamma \times NpL \tag{SM.9}$$

where

N is a volume indicator,
p is the probability of a loss event, and
L is the loss given event for each business line/risk type.

It is recognized in the Basel II proposals that NpL corresponds to the expected annual loss when the loss frequency is binomially distributed and the loss severity is L—severity is not regarded as a random variable in the basic form of the IMA. However, no indication of the possible range for gamma (γ) has been given. Since gamma is not directly related to observable quantities in the annual loss distribution, it is not surprising that the Basel proposals for calibration of gamma were changed. Initially, in their second consultative document (Basel Committee, 2001a), the committee proposed to provide industry-wide gammas, as it has for the alphas in the basic indicator approach and the betas in the standardized approach (see Alexander, 200?). Currently, it is proposed that individual banks will calibrate their own gammas, subject to regulatory approval. **Au: Pls. provide year.**

How should the gammas be calibrated? In this section we show first how (SM.9) may be rewritten in a more specific form, which, instead of gamma, has a new parameter that is denoted phi (ϕ). The advantage of this seemingly innocuous change of notation is that the parameter ϕ has a simple relation to observable quantities in the loss frequency distribution, and therefore ϕ can be calibrated. In fact, we will show that ϕ has quite a limited range: It is bounded below by 3.1 (for very high-frequency risks) and is likely to be less than 4, except for some very low-frequency risks with only one event every four or more years.

We will show how to calculate ϕ from an estimate of the expected loss frequency and that that there is a simple relationship between ϕ and gamma. Table SM.6 gives values for the Basel gamma factors according to the risk frequency. We also consider generalizations of the basic IMA formula (SM.9) to use all the standard frequency distributions, not just the binomial distribution, and include loss severity variability, and show that when loss severity variability is introduced, the gamma (or ϕ) should be reduced.

A Basic Formula for the ORR

Operational risk capital is to cover unexpected annual loss = [99.9th percentile annual loss − mean annual loss], as shown in Figure SM.3. Instead of following CP2.5 and writing unexpected loss as a multiple (γ) of expected loss, we write unexpected loss as a multiple (ϕ) of the loss standard deviation. That is,

$$\text{ORR} = \phi \times \text{standard deviation of annual loss}$$

Since ORR = [99.9th percentile annual loss − mean annual loss], we have

$$\phi = (\text{99.9th percentile} - \text{mean})/\text{standard deviation} \tag{SM.10}$$

in the annual loss distribution.

The basic IMA formula (SM.9) is based on the binomial loss frequency distribution, with no variability in loss severity L. In this case, the standard deviation in loss frequency is $\sqrt{(N\mathrm{p}(1-p))} \approx \sqrt{(Np)}$ because p is small, and the standard deviation in annual loss is therefore $L\sqrt{(Np)}$. Thus, an approximation to (SM.9) is:

$$\text{ORR} = \phi \times L \times \sqrt{(Np)} \qquad \text{(SM.11)}$$

Some points to note about (SM.11) are:

- Equation (SM.10) can be used to calibrate ϕ using the 99.9th percentile, mean, and standard deviation of the frequency distribution because loss severity is not random. The results are given in Table SM.6.
- There is a simple relationship between the original parameter suggested by the Basel Committee in CP2.5 (γ) and ϕ. Indeed, equating (SM.9) and (SM.11) gives

$$\gamma = \phi/\sqrt{(Np)}$$

  We will see that ϕ lies in a narrow range, but there is a much larger range for the values of gamma (see Table SM.6).
- The ORR should increase as the square root of the expected frequency: It will not be linearly related to the size of the bank's operations.
- The ORR is linearly related to loss severity: High-severity risks will therefore attract higher capital charges than low-severity risks.
- The ORR also depends on ϕ, which in turn depends on the dispersion in the frequency distribution. Table SM.6 shows that high-frequency risks, where λ is high, have lower ϕ than low-frequency risks, and therefore they will attract lower capital charges.

Calibration: Normal, Poisson, and Negative Binomial Frequencies

As mentioned above, the basic IMA formula (SM.9) or (SM.11) assumes the binomial distribution (SM.3) for the loss frequency. But there are some important extensions of this framework to be considered. Consider first the approximation to the binomial model for very high-frequency risks, such as those associated with back-office transactions processing. In this case, the binomial distribution (SM.3) may be approximated by the normal distribution—assuming the loss probability is small enough to warrant this. (If it were not, the bank would be facing a risk of such high expected frequency that it should be controlling it as a matter of urgency.) In the normal distribution, the ratio

$$\phi = (\text{99.9th percentile} - \text{mean})/\text{standard deviation} = 3.10$$

as can be found from standard normal tables. We shall see that this provides a lower bound for ϕ.

Consider another frequency distribution, the Poisson distribution (SM.4) with parameter $\lambda = Np$ being the expected number of loss events (per year). The Poisson should be preferred to the binomial frequency distribution if N is difficult to quantify, even as a target. Now (SM.11) may be rewritten

$$\text{ORR} = \phi \times L \times \sqrt{\lambda} \qquad \text{(SM.12)}$$

and (SM.10) becomes $\phi = (\text{99.9th percentile} - \lambda)/\sqrt{\lambda}$, and note that in this case $\gamma = \phi/\sqrt{\lambda}$. The values of ϕ and γ may be obtained using probability tables of the Poisson distribution. (Although all percentiles of the Poisson distribution are by definition integers, we interpolate between integer values to obtain values of ϕ that correspond to 99.9th percentiles in the loss severity distribution on 0, L, 2L, 3L, . . .). The results are given in Table SM.7. For example, in the Poisson distribution with $\lambda = 5$, the standard deviation is $\sqrt{5}$ and the 99.9th percentile is 12.77, so $\phi = (12.77 - 5)/\sqrt{5} = 3.475$; the Poisson ϕ will be smaller than this for higher-frequency risks, tending to the normal ϕ of 3.1 as λ increases. Lower-frequency risks will have more skewed frequency distributions and therefore greater ϕ; for example, in the Poisson with $\lambda = 1$, the 99.9th percentile is 4.868, so $\phi = 3.868$. Table SM.7 gives the values of both ϕ and γ for different risk frequencies from 100 loss events per year down to 0.01 (1 event in 100 years). If there are more than 200 events per year, the normal value of $\phi = 3.1$ should be used.

Table SM.7 Gamma and Phi Values (No Loss Severity Variability)

Lamda	**100**	**50**	**40**	**30**	**20**	**10**
99.9th percentile	131.805	72.751	60.452	47.812	34.714	20.662
Phi	3.180	3.218	3.234	3.252	3.290	3.372
Gamma	0.318	0.455	0.511	0.594	0.736	1.066
Lamda	8	6	5	4	3	2
99.9th percentile	17.630	14.449	12.771	10.956	9.127	7.113
Phi	3.405	3.449	3.475	3.478	3.537	3.615
Gamma	1.204	1.408	1.554	1.739	2.042	2.556
Lamda	1	0.9	0.8	0.7	0.6	0.5
99.9th percentile[a]	4.868	4.551	4.234	3.914	3.584	3.255
Phi	3.868	3.848	3.839	3.841	3.853	3.896
Gamma	3.868	4.056	4.292	4.591	4.974	5.510
Lamda	0.4	0.3	0.2	0.1	0.05	0.01
99.9th percentile[a]	2.908	2.490	2.072	1.421	1.065	0.904
Phi	3.965	3.998	4.187	4.176	4.541	8.940
Gamma	6.269	7.300	9.362	13.205	20.306	89.401

[a]For lambda less than 1, interpolation over both lambda and x has been used to smooth the percentiles; even so small nonmonotonicities arising from the discrete nature of percentiles remain in the estimated values of ϕ.

Table SM.7 shows that ϕ must be in a fairly narrow range: from about 3.2 for medium- to high-frequency risks (20 to 100 loss events per year) to about 3.9 for low frequency risks (1 loss event every 1 or 2 years) and only above 4 for very rare events that may happen only once every five years or so. (When loss severity variability is taken into account, these values should be slightly lower, as we shall see below.) However, the Basel Committee's parameter γ ranges from 0.3 (for high-frequency risks) to 10, or more, for very low-frequency risks.

ORR for Two Risk Types

Suppose 25,000 transactions are processed in a year by a back office, the probability of a failed transaction is 0.04, and the expected loss given that a transaction has failed is \$1,000. Then, $Np = 1{,}000$, the expected annual loss is \$1 million, and the standard deviation of annual loss is \$1,000 × $\sqrt{1{,}000}$ = \$31,622.

The loss frequency is binomial distributed, with large N and small p and can therefore be approximated by the normal distribution. In this case, we have shown that $\phi \approx 3.1$ so that the ORR $\approx$ \$(3.1 × 31,622) $\approx$ \$98,000.

On the other hand, if 50 investment banking deals are done in one year, the probability of an unauthorized or illegal deal is 0.005, and the expected loss if a deal is unauthorized or illegal is \$4 million, then $Np = 0.25$ and the expected annual loss will also be \$1 million.

Although the expected loss is the same for both types of risk, the ORR is quite different. The standard deviation of annual loss in investment banking is \$4 million × $\sqrt{0.25} = \$2$ million. The loss frequency is assumed to be Poisson distribution with parameter 0.25. The mean and standard deviation of this distribution are 0.25 and 0.5, respectively, and from Poisson tables, the 99.9th percentile is approximately 2.28, so the ratio $\phi \approx (2.28 - 0.25)/0.5 \approx 4$.

Thus, in investment banking, the unexpected loss (ORR) $\approx$ \$(4 × 2 million) $\approx$ \$8 million. This is almost 100 times greater than the unexpected loss in transactions processing, although the expected loss is the same in both!

In the Poisson distribution, all moments are closely related because there is only one parameter. For example, the mean is equal to the variance, and the higher moments may be obtained using a recursive formula, also depending on λ. In the negative binomial distribution (SM.5), there are two parameters and therefore more flexibility to accommodate difference between the mean and the variance and exceptional skewness or heavy tails.

The negative binomial model also captures the uncertainty in loss probability: It may be viewed as a probability weighted sum of Poisson distributions, each with a different expected loss frequency. The negative binomial density function is given in (SM.5). It has mean $\alpha\beta$ and standard deviation $\beta\sqrt{\alpha}$, so the IMA formula for the ORR (SM.10) becomes

$$\text{ORR} = \phi \times \beta\sqrt{\alpha} \times L \qquad \text{(SM.13)}$$

where $\phi = (99.9\text{th percentile} - \alpha\beta)/\beta\sqrt{\alpha}$.

Again, values for ϕ and $\gamma = \phi/\sqrt{a}$ may be calculated from statistical tables of the negative binomial density function, for different values of α and β.

The ORR with Random Severity

Up to this point, our discussion of the IMA has assumed that loss severity L was not random. Now suppose that it is random, having mean μ_L and standard deviation σ_L, but that severity is independent of loss frequency. Again denote by p the loss probability in the annual frequency distribution, so that the expected number of loss events during one year is Np, and we assume no uncertainty in loss probability. At each of the N events there is a constant probability p of a loss event, in which case the loss severity is random. The expected annual loss X has moments:

$$E(X) = NpE(L) = Np\mu_L$$
$$E(X^2) = NpE(L^2) = Np\left(\mu_L^2 + \sigma_L^2\right)$$

Therefore, the annual loss variance is

$$\text{Var}(X) = Np\left(\mu_L^2 + \sigma_L^2\right) - (Np\mu_L)^2 \approx Np\left(\mu_L^2 + \sigma_L^2\right)$$

because p is small. More generally, writing $\lambda = Np$, the expected loss frequency in the Poisson model, the annual loss X has variance

$$\text{Var}(X) \approx \lambda\left(\mu_L^2 + \sigma_L^2\right)$$

and the IMA capital charge (SM.10) is therefore:

$$\text{ORR} = \phi \times \mu_L \times \sqrt{\lambda} \times \sqrt{(1 + (\sigma_L/\mu_L)^2)} \qquad \text{(SM.14)}$$

Note that when the loss severity is random, the calibration parameter ϕ refers to the annual loss distribution, and not just the frequency distribution. With the above notation:

$$\phi = (99.9\text{th percentile of annual loss} - \lambda\mu_L)/\sqrt{\left[\lambda\left(\mu_L^2 + \sigma_L^2\right)\right]}$$

and this reduces to the previous formula for ϕ when $\sigma_L = 0$, since in that case the 99.9th percentile of annual loss was equal to the 99.9th percentile of frequency × μ_L. Note that when $\sigma_L \neq 0$, ϕ should be somewhat *less* than the frequency based ϕ that has been tabulated in Table SM.6, because the annual distribution will tend to be less skewed than the frequency and severity distribution, but ϕ will still be bounded below by the value of 3.1, which corresponds to the normal annual loss distribution. Recall that in Table SM.6 the value of ϕ ranged from about 3.2 for medium to high frequency risks to around 4 for low frequency risks, and only for rare events would it be greater than 4. By how much should ϕ be reduced to account for loss severity variability? We address this question by way of an example earlier in this chapter.

How is the Basel "gamma" affected by the introduction of loss severity variability? Since now

$$\gamma = \phi\sqrt{(1 + (\sigma_L/\mu_L)^2)}/\sqrt{\lambda}$$

the Basel parameter is likely to be much greater than that given in Table SM.6, particularly if σ_L is large.

Comparison of (SM.14) with (SM.11) shows that when there is uncertainty in loss severity, an extra term $\sqrt{(1 + (\sigma_L/\mu_L)^2)}$ should be used in the ORR formula. Thus, the greater the uncertainty in loss severity, the greater the capital charge. This term is likely to be close to one for high-frequency risks that have little variation in loss severity, but it may be greater for low-frequency risks, where loss severity variability may a similar order of magnitude to the expected loss severity. In the case that $\sigma_L = \mu_L$, the ORR should be multiplied by $\sqrt{2}$, and if $\sigma_L > \mu_L$ there will be an even greater increase in the ORR.

It is not only the type of risk that determines the magnitude of $\sqrt{(1 + (\sigma_L/\mu_L)^2)}$. The method for estimating the parameters σ_L and μ_L will also play an important role. Recall from the example given in Figure SM.3 that only Bayesian estimation of loss severity parameters can properly recognize subjectivity, and different sources of data. When both "hard" internal loss experience data and soft data, from an external consortium, or a scorecard, are to be combined, it is essential to use Bayesian estimates rather than the maximum likelihood estimates. The next example

Au: Pls. provide table title.

Table SM.8

	Bayesian	Classical
μ_L	2.31	2.71
σ_L	0.83	1.43
p	0.061	0.058
N (target)	100	100
ϕ	3.45	3.45
ORR (7.11)	19.61	22.60
ORR (7.14)	20.85	25.55

uses the formulas developed in this section, with a fixed set of internal and external data, and compares the capital charge when (a) classical and (b) Bayesian estimators are employed.

Comparison of Bayesian and Classical Estimates of ORR

In examples provided in Figures SM.3 and SM.4 the Bayesian and classical estimates of the mean loss severity, μ_L and the standard deviation of loss severity, σ_L and the loss probability p were compared. We used the internal and external data from Figure SM.3. Now, using the formulas (SM.11) and (SM.14), we compute the ORR, with and without loss severity uncertainty, and compare the difference in the ORR when using Bayesian versus classical estimation of the parameters. The results are shown in Table SM.8.

Without loss severity uncertainty, the formula (SM.11) is used for the ORR and the calibration of ϕ used the Poisson frequency density with parameter $\lambda = Np \approx 6$, giving $\phi = 3.45$ from Table SM.6. In this case the classical estimate of the ORR is 22.60, which is about 15% higher than the Bayesian estimate. The introduction of severity uncertainty, and consequently the use of (SM.14) for the ORR, increases this difference: The classical estimate of the ORR increases to 25.55, which is now 22.5% larger than the Bayesian estimate of the ORR. [Note that we used the same ϕ in (SM.14), although a slightly lower value is appropriate when $s_L \neq 0$, as mentioned earlier. However, this does not affect the relative magnitude of the Bayesian and classical estimates of the ORR.]

Inclusion of Insurance and the General Formula

The Basel Committee (2001b) states that banks will be permitted to reduce capital charges to allow for insurance cover only if they use an advanced measurement approach. Their justification is that "this reflects the quality of risk identification, measurement, monitoring and control inherent in the AMA and the difficulties in establishing a rigorous mechanism for recognizing insurance where banks use a simpler regulatory capital calculation technique." Banks that mitigate certain operational risks through insurance will, hopefully, regard this "carrot" as an extra incentive to invest in the data and technology required by the AMA. They will also need to develop an appropriate formula for recognition of insurance that is "risk-sensitive but not excessively complex," in the words of the Basel Committee.

A simple formula for including insurance coverage in the operational risk charge can be deduced using the binomial model. Insurance reduces the loss amount when the event occurs (an expected amount μ_R is recovered) but introduces a premium C to be paid even if the event does not occur. An expected amount $\mu_L - \mu_R$ is lost with probability p and C is lost with probability 1, so the expected annual loss is now $N[p(\mu_L - \mu_R) + C]$. If we assume that the premium is fairly priced, then the introduction of insurance will not affect the expected loss significantly. Thus, the expected loss will be approximately $Np\mu_L$ as it was before the insurance, and this will be the case if the premium is set to be approximately equal to the expected payout, that is, $C \approx p\mu_R$. However, insurance will reduce the standard deviation of annual loss and therefore also the capital charge.

Assuming p is small, the annual loss standard deviation will now be approximately $\sqrt{(Np)} \times (\mu_L - \mu_R) \times \sqrt{(1 + (\sigma_L/\mu_L)^2)}$. Denote the expected recovery rate by r, so that $r = \mu_R/\mu_L$ and set $Np = \lambda$ as usual. Then (SM.14) becomes

$$\text{ORR} = \phi \times \sqrt{\lambda} \times \mu_L \times \sqrt{(1 + (\sigma_L/\mu_L)^2)} \times (1 - r)$$

As before, this can be generalized to other types of distributions for loss frequency (in which case $\sqrt{\lambda}$ should be replaced by the standard deviation of the loss frequency distribution). The general result is the same in each case: If risks are insured and the expected recovery rate per claim is r, the capital charge should be reduced by a factor of $(1 - r)$. The general formula for the ORR is thus:

$$\text{ORR} = \phi \times \sigma_F \times \mu_L \times \sqrt{(1 + (\sigma_L/\mu_L)^2)} \times (1 - r) \quad \text{(SM.15)}$$

where σ_F is the standard deviation of the frequency distribution. Of course, insurance is more complex than this because contracts will not cover individual events except perhaps for very large potential losses. However, it is stated in Basel Committee (2001b) that a simple formula, such as (SM.15), will be necessary for banks that wish to allow for insurance coverage when calculating capital charges.

SIMULATING THE ANNUAL LOSS DISTRIBUTION

For each risk type/line of business, the annual loss distribution is the compound distribution of the loss frequency and loss severity, as in (SM.2) and illustrated in Figure SM.4. A simple simulation algorithm based on (SM.2) may be used to generate an annual loss distribution as follows:

1. Take a random draw from the frequency distribution: Suppose this simulates n loss events per year.
2. Take n random draws from the severity distribution: Denote these simulated losses by $L_1, L_2, \ldots L_n$.

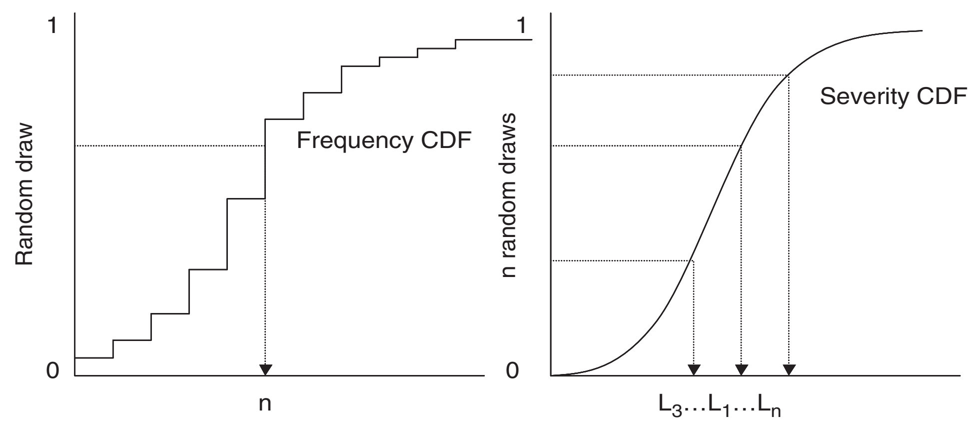

Figure SM.6: Simulating the Annual Loss Distribution

3. Sum the *n* simulated losses to obtain an annual loss $X = L_1 + L_2 + \ldots + L_n$.
4. Return to step 1, and repeat several thousand times: thus obtain $X_1, \ldots X_N$ where N is a very large number.
5. Form the histogram of $X_1, \ldots X_N$: this represents the simulated annual loss distribution.
6. The ORR for this risk type/line of business is then the difference between the 99.9*th* percentile and the mean of the simulated annual loss distribution.

Figure SM.6 illustrates the first two steps in the simulation algorithm. The use of empirical frequency and severity distributions is not advised, even if sufficient data are available to generate these distributions empirically. There are two reasons for this. First, the simulated annual loss distribution will not be an accurate representation if the same frequencies and severities are repeatedly sampled. Second, there will be no ability for scenario analysis in the model, unless one specifies and fits the parameters of a functional form for the severity and frequency distributions. Some useful functional forms were discussed earlier in this chapter.

The following example shows that the ORR that is obtained through simulation of the annual loss distribution is approximately the same as that which is estimated through an analytic approximation.

Comparison of ORR from Analytic and Simulation Approximations

Suppose the frequency is Poisson distributed with parameter $\lambda = 5$, so the expected number of loss events per year is 5. Suppose the severity is gamma distributed with $\alpha = 4$ and $\beta = 2$, so that the mean severity $\mu_L = \$8$ million and the standard deviation $\sigma_L = \$4$ million. Thus, the expected annual loss is \$40 million.

1. Estimating the ORR using formula (SM.14), with $\lambda = 5$, $\mu_L = 8$, $\sigma_L = 4$.

   From our previous discussion we know that ϕ has a lower limit of 3.1 and an upper limit that depends on λ and which is given in Table SM.6. From that table, when $\lambda = 5$, the upper limit for ϕ is 3.47. Now $\phi = 3.47$ gives the analytic approximation for the ORR = \$69.4 million, and $\phi = 3.1$, gives the analytic approximation for the ORR = \$62 million. The ORR will be between these two values, in the region of \$64 million ($\phi = 3.2$) to \$66 million ($\phi = 3.3$).
2. Simulating the ORR using Poisson frequency with $\lambda = 5$ and gamma severity with $\alpha = 4$ $\beta = 2$.

   In Excel, 5,000 frequency simulations and the requisite number of severity simulation for each were performed using the random number generator function for the Poisson and the formula "=(GAMMAINV(RAND(),4,2)" and according to the compound distribution algorithm described above. In this way, 5,000 annual losses were simulated and the ORR was estimated as the difference between the 99.9th percentile and the mean of the simulated distribution. The estimate obtained was \$64.3 million.

AGGREGATION AND THE TOTAL LOSS DISTRIBUTION

The *aggregation* of the ORR over all risk types and lines of business, to obtain a total ORR for the bank, can take into account the likely dependencies between various operational risks. The Basel Committee (2001b) states: "The bank will be permitted to recognize empirical correlations in operational risk losses across business lines and event types, provided that it can demonstrate that its systems for measuring correlations are sound and implemented with integrity." In this section, we first consider the aggregation to a total unexpected annual loss for the bank when the analytic approximation (the IMA) is used for each unexpected annual loss. We show how to account for correlations in this aggregation. Then we consider the more complex problem of aggregating the individual annual loss distributions that are estimated using a loss model, into a total annual loss distribution for the bank.

Aggregation of Analytic Approximations to the ORR

Recall that when unexpected loss is estimated analytically, as described earlier, for each line of business ($i = 1, 2, \ldots n$) and risk type ($j = 1, 2, \ldots, m$) we have

$$\text{ORR}_{ij} = \phi_{ij}\sigma_{ij}$$

where σ_{ij} is the standard deviation of the annual loss distribution.

Two simple methods for obtaining the total ORR for the bank are:

1. Sum these ORR_{ij} over all line-of-business and risk types.
2. Take the square root of the sum of squares of the ORR_{ij} over all line-of-business and risk types.

The simple summation (1) assumes perfect correlation between the annual losses made in different line-of-business and risk types. This remark follows from the observation that the standard deviation of a sum of random variables is equal to the sum of their standard deviations only if their correlations are in unity. If the multipliers ϕ_{ij} were all the same, and if all dependency between annual loss distributions were measured by their correlations, we could conclude that the summation of operational risk capital charges assumes that risks are perfectly correlated. This implies that all operational loss events must occur at the same time, which is totally unrealistic. The summation (2) assumes zero correlation between operational risks, which occurs when they are independent. Again, this assumption is not very realistic and it will tend to underestimate the total unexpected loss, as shown by Frachot, Georges, and Roncalli (2001).

More generally, suppose that we have a correlation matrix **V** that represents the correlations between different operational risks—this is a heroic assumption, about which we shall say more later on in this section. Nevertheless, suppose the $(n + m) \times (n + m)$ correlation matrix **V** is given. We have the $(n + m) \times (n + m)$ diagonal matrix **D** of standard deviations σ_{ij}, that is $\mathbf{D} = \text{diag}(\sigma_{11}, \sigma_{12}, \sigma_{13}, \ldots \sigma_{21}, \sigma_{22}, \sigma_{23}, \ldots, \ldots \sigma_{nm},)$ and the $(n + m)$ vector ϕ of multipliers. Now the total unexpected loss, accounting for the correlations given in **V** is $\text{Sqrt}(\phi'\mathbf{DVD}\,\phi)$.

Comments on Correlation and Dependency

One of the advantages of the simulation approach is that the whole annual loss distribution is estimated for each type of risk, and not just the unexpected loss. In this case, it is possible to account for dependencies other than correlations when combining these distributions to obtain the total annual loss distribution. Correlation, which is a standardized form of the first moment of the joint density of two random variables, is not necessarily a good measure of the dependence between two random variables. Correlation only captures linear dependence, and even in liquid financial markets, correlations can be very unstable over time. In operational risks is it more meaningful to consider general codependencies, rather than restrict the relationships between losses to simple correlation. An example of a possible dependency structure that may be determined by some key risk indicators, is given in Table SM.9.

Modeling codependency between frequencies is indeed a primary issue, following the observation that operational losses may be grouped in time, rather than by severity. Frachot, Georges, and Roncalli (2001) advocate the use of a multivariate extension of the Poisson distribution to model correlated loss frequencies. However, the approach is tractable only for the aggregation of two frequency distributions. In this section we consider how to model dependency between the annual loss distributions, rather than just the dependency between loss frequencies. It should be noted that loss severities may also be codependent, since operational loss amounts can be affected by the same macroeconomic variable (e.g., an exchange rate).

It should also be noted that the most important dependency is not the dependency between one operational loss and another—it is between the costs and revenues of a particular activity. Operational risks are mostly on the cost side, whereas the revenue side is associated with market and/or credit risks. In fact, vertical dependencies, between a given operational risk and the market and/or credit risks associated with that activity, are by far the most important dependencies to account for when estimating the total risk of the bank. The effect of accounting for dependencies between different operational risks will be substantial, as shown in the example given later in this chapter. However, this effect will be marginal compared to the effect of accounting for dependencies between operational, market, and credit risks. Indeed, from the point

Table SM.9 Dependence among Operational Risks

Risk	Downsizing of Back-Office Personnel	Expansion of Business in Complex Products	Outsource and Improve Systems and IT	Overall Effect
1. Internal Fraud	+	+	−	+
2. External Fraud	+	0	−	0
3. Employment Practices and Workplace Safety	+	0	+	+
4. Clients, Products, and Business Practices	0	+	0	+
5. Damage to Physical Assets	0	0	0	0
6. Business Disruption and System Failures	0	+	−	0
7. Execution, Delivery, and Process Management	+	+	+	+

of view of economic capital within the enterprise-wide framework, operational risks should be negligible compared with the other two risks, unless one needs to consider extremely rare, large-impact events.

The Aggregation Algorithm

We now consider how the distribution of the total annual loss is obtained from the distributions of individual annual losses. The method proposed here is to sum losses in pairs where for each pair, a *copula* is chosen to define a suitable dependency structure. Some details on copulas are given in the appendix to this chapter. The algorithm consists of two steps, which are first explained for aggregating two annual losses X and Y. Then we comment on the extension of the algorithm to more than two annual losses:

Step 1: Find the joint density $h(x,y)$ given the marginal densities $f(x)$ and $g(y)$ and a given dependency structure:

If X and Y were independent then $h(x, y) = f(x)g(y)$. When they are not independent, and their dependency is captured by a copula, then

$$h(x, y) = f(x)g(y)c(x, y) \qquad \text{(SM.16)}$$

where $c(x,y)$ is the probability density function of the copula.

Step 2: Derive the distribution of the sum $X + Y$ from the joint density $h(x, y)$:

Let $Z = X + Y$. Then the probability density of Z is the "convolution sum"

$$k(z) = \sum_x h(x, z - x) = \sum_y h(z - y, y)$$

if $h(x,y)$ is discrete, or if $h(x,y)$ is continuous, the "convolution integral"

$$k(z) = \int_x h(x, z - x)dx = \int_y h(z - y, y)dy$$

Now suppose there are three annual losses X, Y, and Z with densities $f_1(x)$, $f_2(y)$, and $f_3(z)$, and suppose that X and Y have a positive dependency but Z is independent of both of these. Then we aggregate in pairs as follows:

1. Using $f_1(\mathbf{x})$ and $f_2(y)$, we obtain the joint density $h_1(x,y)$ of X and Y, and this requires the use of a copula that captures the positive dependency between X and Y.
2. Then we use "convolution" on $h_1(x, y)$ to calculate the density $k(w)$ of $W = X + Y$;
3. By independence the joint density of W and Z is $h_2(w, z) = k(w)f_3(z)$.
4. Using the convolution on $h_2(w,z)$, we obtain the density of the sum $X + Y + Z$.

The algorithm can be applied to find the sum of any number of random variables: If we denote by X_{ij} the random variable that is the annual loss of the line of business (i) and risk type (j), the total annual loss has the density of the random variable $X = \sum_{i,j} X_{ij}$. The distribution of X is obtained by first using steps 1 and 2 of the algorithm to obtain the distribution of $X_{11} + X_{12} = Y_1$, say, then these steps are repeated to obtain the distribution of $Y_1 + X_{13} = Y_2$, say, and so on.

Aggregation of Annual Loss Distributions under Different Dependency Assumptions

The preceding shows that dependency structures that are more general than correlation may also be used for aggregating distributions, simply by choosing the appropriate copula to generate the joint density in (SM.16). A good approximation to the joint density is

$$h(x, y) = f(x)g(y)c(J_1(x), J_2(y))$$

where the standard normal variables J_1 and J_2 are defined by:

$$J_1(x) = \Phi^{-1}(F(x)) \quad \text{and} \quad J_2(y) = \Phi^{-1}(G(x))$$

where Φ is the standard normal distribution function, F and G are the distributions functions of X and Y, and

$$\begin{aligned} c(J_1(x), J_2(y)) = \exp\{&-[J_1(x)^2 + J_2(y)^2 \\ &- 2\rho J_1(x)J_2(y)]/2(1-\rho^2)\}\exp\{[J_1(x)^2 \\ &+ J_2(y)^2]/2\}/\surd(1-\rho^2) \end{aligned} \qquad \text{(SM.17)}$$

This is the density of the Gaussian copula. [This was first shown by Nataf (1962); Mardia (1970) provides sufficient conditions for $f(x,y)$ to be a joint density function when $c(J_1(x), J_2(y))$ is given by (SM.17).]

The Gaussian copula can capture positive, negative, or zero correlation between X and Y. In the case of zero correlation, $c(J_1(x), J_2(y)) = 1$ for all x and y. Note that annual losses do not need to be normally distributed for us to aggregate them using the Gaussian copula. However, a limitation of the Gaussian copula is that dependence is determined by correlation and is therefore symmetric.

Many other copulas are available for dependency structures that are more general than correlation. For example, a useful copula for capturing asymmetric tail dependence is the Gumbel copula. An illustration of the aggregation algorithm is provided in Alexander (200?). **Au: Pls. provide year.**

Specifying Dependencies

How should a bank specify the dependence structure between different operational risks? If it seeks to include correlations in the (IMA) analytic approximation to unexpected loss, then it needs a correlation matrix $\mathbf{V}$ between all the different operational risks that it faces, over all business lines and risk types. Given the remarks already made about correlations of operational risks, attempting to calibrate such a matrix to any sort of data would be very misleading indeed.

A more realistic exercise is to link the dependencies between operational risks to the likely movements in common attributes. Examples of key risk drivers are volume of transactions processed, product complexity, and staffing (decision) variables such as pay, training, recruitment, and

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so forth. [See Alexander (200? or the central ideas and illustrations.) It is the assumption that risk drivers may be linked to the dependencies between operational risks. Rather than attempting to specify a correlation between each and every operational risk, over all business lines and risk types, a better alternative approach is to examine the impact that likely changes in key risk drivers will have on different categories of operational risks.

Knowing the management policies that are targeted for the next year, a bank should identify the likely changes in key risk drivers resulting from these management decisions. In this way, the probable dependence structures across different risk types and lines of business can be identified. For example, suppose two operational risks are thought to be positively dependent because the same risk drivers tend to increase both of these risks and the same risk drivers tend to decrease both of these risks. In that case, we should use a copula with positive dependency for aggregating to the total annual loss distribution. We further these ideas by example. Table SM.9 considers the impact of three management policies on the seven risk types that are defined by the Basel Committee, for a fixed line of business. The entries +, 0, and − imply that the policy is likely to increase, have no effect on, or decrease the operational risk.

If a bank were to rationalize the back office, with many people being made redundant, this would affect risk drivers such as transactions volume, staff levels, skill levels, and so forth. The consequent difficulties with terminations, employee relations, and possible discriminatory actions would increase the Employment Practices and Workplace Safety risk. The reduction in personnel in the back office could lead to an increased risk of Internal Fraud and External Fraud, since fewer checks would be made on transactions, and there may be more errors in Execution, Delivery, and Process Management. The other risk types are likely to be unaffected.

Suppose the bank expands its business in complex products, perhaps introducing a new team of quantitative analysts. Internal Fraud could become more likely and potentially more severe. Model errors, product defects, aggressive selling, and other activities in the Clients, Products, and Business Practices category may increase in both frequency and severity. Business Disruption and System Failures will become more likely with the new and more complex systems. Finally, there are many ways in which Execution, Delivery, and Process Management risk would increase, including less adequate documentation, more communication errors, collateral management failures, and so forth.

Finally, suppose the bank decides to outsource its systems and information technology (IT), hopefully to improve it. This should have a positive effect on risk drivers such as systems downtime, so Business Disruption and System Failures should become less risky. IT skill levels should be increased so that Internal Fraud and External Fraud would become more difficult. But this policy could increase risk in Execution, Delivery, and Process Management, due to communications problems with an external firm having different systems. Also, there would be a negative effect on staff levels, and termination of contracts with the present IT and systems personnel may lead to employee relation difficulties and thus increase the Employment Practices and Workplace Safety risk.

It may be that these three policies are only some of those under consideration by management, but if they are the only foreseeable changes in management due for implementation during the next year, the likely net effect is shown in the last column of Table SM.9. This would imply that, for aggregating risks 1, 3, 4, and 7, copulas with positive dependence should be used. The weaker the codependency denoted by the "+" sign in the last column of the table, the smaller the value of the dependency parameter. Then, to aggregate these with the other risks, an independence assumption for the joint densities would be appropriate.

An advantage of this methodology is that operational risk capital and operational risk dependence can be assessed at the scenario level. That is, management may ask, "What would be the net effect on operational risk capital, if a key risk drivers (e.g., product complexity) is increased?" Thus, it provides a means whereby the economic capital and the minimum regulatory capital requirement for the bank can be assessed under different management policies.

SUMMARY

The main focus of this chapter was to give a pedagogical introduction to the statistical/actuarial approach of modeling frequency and severity distributions, with many illustrative examples. From the outset, Bayesian estimation methods are shown to be the natural approach to operational loss parameter estimation, rather than maximum likelihood or other classical techniques such as method of moment estimation. This is because of the high level of subjectivity in operational loss data, whether it be from scorecards or from an external data consortium. We showed how to obtain Bayesian estimates of loss probabilities, and of loss severity means and standard deviations, and we have considered the effect on capital charges of using Bayesian rather than classical estimation.

This chapter examined the Advanced Measurement Approach that was suggested by the Basel Committee (2001b) in much detail. In contrast to the impression given by the Basel Committee, there is really only one approach to estimating operational risk, and that is the actuarial approach. That is, the foundation of any advanced measurement model rests on the compounding of frequency and severity distributions. An example is given to show that the analytic approximation to unexpected loss (the IMA formula) is very close to the unexpected loss that is estimated by simulating the annual loss distribution (the LDA method). A useful references table of the Basel "gamma" factors was provided, and various extensions of the basic IMA formula were derived.

Then the use of copulas for aggregating operational risks was explained, as well as how the correlation—or, more generally, the codependency—among operational risks will have a great impact on the aggregate unexpected loss.

Throughout this chapter we have commented that it is misguided to use the 99.9th percentile to estimate operational risk, given the uncertainty about the form of frequency and severity distributions, the subjectivity of data, the imprecision of parameter estimates, and, most of all, the difficulty in capturing their dependencies when aggregating to the total loss distribution.

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