There are three reasons to trade commodity options: diversification, hedging, and speculation. Options are included in investment portfolios because they have a limited upside or downside, compared with futures. Commodity options provide diversification because they have low correlations with equities and bonds. For that reason it is optimal to diversify by adding commodity options to standard portfolios despite their being risky instruments.

Risk managers use commodity options to hedge price risk. For instance, calendar spreads can be used to protect producers in a market that tends to swing between backwardation and contango. Average price options (where the payoff depends on the difference between some average of underlying prices and the option strike) are also popular for risk management because they are much cheaper than standard options—yet they still allow the purchaser to secure supplies at a fixed price.

Speculators use options as highly leveraged bets on price direction. For instance, a U.S. calendar spread call on the difference between the one-month futures price and the three-month futures price is a bet that futures will move to stronger backwardation at some time before the option’s expiry. At exercise the purchaser receives a long position on the one-month
futures and a short position on the three-month futures, at their prevailing market prices.

To buy an option is to be long volatility. Hence commodity options can also be used to speculate on volatility and to hedge volatility risk. All commodity prices are volatile, some more than others. Agriculturals tend to have the lowest volatilities, generally only around 30% to 50% but metals and energy have much higher volatility. For instance, the volatility of on-peak spot electricity prices in the United States was almost 200% in 2005.

This chapter provides a survey of the market for commodity options, the products that are commonly traded and the models that we can use to price and hedge commodity options.

**COMMODITY OPTIONS MARKETS**

The volume of exchange-traded options on commodities has grown steadily since the first contracts were introduced in the late 1980s. NYMEX and COMEX are the most active platforms for trading, mostly in different types of U.S. options on energy and metals futures. In 2006 a total of 60 million commodity options contracts were traded on the Nymex and Comex exchanges, over 25% of the total volume traded on commodity futures contracts (see Exhibit 24.1).

![Options and Futures Trading Volume](chart.png)

**EXHIBIT 24.1** NYMEX Futures and Options Trading Volume
There is much more exchange trading on energy options than on metal options. Exhibit 24.2 shows that the most liquid energy options on NYMEX are American options on crude oil and natural gas. American crack spread options have traded for many years but trading volume remains low. American calendar spread options and average price options have grown more popular during the last few years (see Exhibit 24.3). Calendar spreads and average price options provide flexibility for the risk management of commodity futures and, since they are cheaper than standard options, increase the potential for speculation.

The more liquid energy contracts also have cash-settled European-style options contracts that are traded on exchanges: daily options available solely for clearing and inventory options that help manage exposure to the impact of reported inventories. The strike units of inventory options are the potential difference in inventory from the previous week’s report, the change in the inventories determine which options are in-the-money and which are out-of-the-money, and the premium collected from those holding out-of-the-money options is paid to those holding in-the-money options. As well as vanilla options, digital inventory options pay a fixed amount for in-the-money contracts.

Exhibit 24.4 shows that most of the options traded on COMEX are standard American gold and silver futures options, although average daily volumes on copper futures options are also significant.
**EXHIBIT 24.3** Energy Spread Options Trading Volume

**EXHIBIT 24.4** COMEX Metal Options Trading Volume
Two other large exchanges specialize in options on futures of agricul-
turals such as dairy products, cocoa, coffee, sugar, soybean products, corn, 
wheat, live cattle, and lean hogs. These are the CME group (created by tac 
merger of Chicago Mercantile Exchange (CME) and the Chicago Board of 
Trade (CBOT)) and the main commodity options exchange outside the U.S., 
Euronext.Liffe.

Hence the major commodities exchanges all trade in options on futures 
of the same maturity as the option, and not in options on spot prices. There 
are two good reasons for this. First, the no-arbitrage argument that gives the 
option price has to be based on the possibility of hedging with a liquid asset, 
and the futures are usually far more liquid than the spot. Secondly, spot 
prices are more difficult to model than futures prices because they display 
mean reversion that is related to seasonally and long-term economic equilib-
ria that equate supply and demand. (For instance, if a Chinese car manufac-
turer dramatically increases production of inexpensive cars, the price of 
gasoline will increase in the short term but over the longer term more refin-
eries would be built to meet the demand for gasoline.) In contrast, a fixed-
term futures contract is a martingale under the risk neutral measure.

A variety of commodity options are traded over-the-counter (OTC) and 
for these the underlying can be the spot price rather than the futures price. 
Common products include caps (which provide upside protection), floors 
(which provide downside protection), and collars (which provide both). 
Path dependent options such as average price options and barriers, which 
are cheaper than standard options, are also traded OTC.

A particularly risky OTC option is the floating strike option. The holder 
of a floating strike European call contract has the right, but not the obliga-
tion, to buy the commodity at the strike price every day during the exercise 
month. The strike price is based on the previous end-of-month price. The 
price of the commodity could change considerably during the exercise month, 
hence the writers of such options face huge risks. These products are also 
difficult to hedge and so are rather expensive. Nevertheless, the demand for 
such products is considerable and even more complex and risky products 
such as have recently become popular.

**HISTORICAL PRICE BEHAVIOR**

This section examines the behavior of daily spot and futures prices during 
2005 and 2006 in five different commodities that have actively traded fu-
tures options on U.S. exchanges. These are corn, lean hogs, silver, natural 
gas, and electricity. They have been chosen to represent the three main 
classes of commodities: agricultural, metals, and energy. We demonstrate
that the price processes for these five different commodities are remarkably different.

**Corn**

Exhibit 24.5 depicts the spot price on the right hand scale and several futures prices on the left hand scale. Throughout most of 2005 and 2006, the market was in contango and futures of different maturities are highly correlated with each other and with the spot price. Prices can jump at the time of the U.S. Department of Agriculture crop production forecasts and in response to news announcements. A recent example of this was the reaction to President Bush’s announcement of plans to increase ethanol production, clearly visible in January 2007.

**Lean Hogs**

Exhibit 24.6 shows the spot and futures prices of lean hogs. Futures prices display low correlation across different maturities with winter futures prices being noticeably lower than summer futures prices. The market is
characterized by a relatively flat demand and an inelastic supply that is set by the farmer’s decision to breed 10-months previously. High prices induce producers to retain more sows for breeding. This pushes the price even higher—and prices tend to peak in the summer months when supply of live hogs is usually at its lowest. Price jumps may correspond to the U.S. Department of Agriculture official “hogs and pigs” report on the size of the breeding herd.

Silver

Exhibit 24.7 shows the spot and futures prices of silver. Silver futures contracts are actively traded on NYMEX for every month and only a selection of months is shown in Exhibit 24.7. The market is narrower than the gold market because there are less reserves of silver. On the demand side, silver is used in industrial processes (e.g., silver plating and electronics); but there is no inherent seasonality in these. Hence, the term structure is very highly correlated indeed, basis risk is small, and prices display no seasonality. The frequent spikes and jumps are the result of speculative trading.
Natural Gas

Exhibit 24.8 shows the spot and futures prices of natural gas. Natural gas futures are less highly correlated than oil futures prices, and swings between backwardation and contango are seasonal. Backwardation tends to occur during winter months when short-term futures prices can jump upward. Contango is more likely during summer months. There is a large basis risk with spot price spikes arising during unexpected cold snaps. Down spikes may also occur in the summer when storage is full to capacity.

Electricity

Exhibit 24.9 illustrates the prices of PJM spot electricity and associated futures contracts (data were only available for 2005). Since electricity cannot be stored, spot prices are excessively variable and rapidly mean reverting, especially during summer months when the air conditioning required during heat waves increases demand. The term structure on six different days during the sample period is shown in Exhibit 24.10. This is very different to the term structure of other commodity futures prices. The general level of futures prices is lowest in the spring (March to May) and highest in the winter.
EXHIBIT 24.8  Natural Gas Futures and Spot Prices
Source: Data from Bloomberg.
*aHenry Hub Natural Gas Spot and Henry Hub Natural Gas Futures prices (USD/MMBtu).

EXHIBIT 24.9  Electricity Futures and Spot Prices
Source: Data from Bloomberg.
*aReal Time LMP Electricity Spot Price Western P-J-M and P-J-M Electricity Futures (USD/MwH).
(November to January) and the futures that expire in July, August, January, and February have the highest prices.

**STOCHASTIC PROCESSES**

The prices of liquid exchange-traded commodity options will be set by market makers responding to demand and supply. However, some exchange-traded commodity options are highly illiquid (e.g., aluminum futures options on NYMEX). Also, many commodity options trades are over the counter. To price such options traders need to specify a stochastic process for the underlying (spot or futures) price.

Option pricing models have parameters that should be calibrated to liquid market prices of associated options, such as exchange-traded standard call and put options. This is to avoid arbitraging a trader’s price by replicating its payoff using the liquid options. For instance the price of a calendar spread option on crude oil futures should be consistent with the market prices of the American crude oil futures options in the two legs of the spread.

The stochastic process should provide a tractable solution for the prices of vanilla options, as this considerably simplifies the calibration to market
data. Often approximate analytic prices of American and European vanilla options are available, but if not, at least these models are amenable to numerical methods of resolution.\(^1\)

**Comparison of Processes for Spot and Futures Prices**

Seasonal patterns and mean reversion are often evident in spot prices and can give rise to term structures of futures that fluctuate between backwardation and contango. But it is important to note that there is no seasonality or mean reversion in the price of any given fixed term futures.

Indeed, every fixed term futures price is a martingale, irrespective of whether the commodity is an investment asset or a consumable asset. Since the contract is virtually costless to trade, its risk-neutral expected value tomorrow should be its value today. Otherwise, all investors could expect to profit from buying the futures (if the expectation is for the price to rise) or from selling the futures (if the expectation is for the price to fall).

Most traded commodity options are options on futures. Moreover, the price of any option on a spot price can be obtained from the futures price process, provided only that the payoff is path independent. At expiry, the spot and futures prices are equal, so they have the same distribution. But the price of a path independent option depends only on the underlying price distribution at expiry. Hence, it makes no difference whether we use the spot price or the futures price to value such an option. We conclude that it is only in the special case of an OTC path dependent option on the spot that we need to specify the spot price process. In the vast majority of cases, therefore, the option price can be based on a martingale process for the futures.

**Geometric Brownian Motion**

*Geometric Brownian motion* (GBM) models are widely used due to their simplicity and flexibility. Under GBM the price has a lognormal distribution, or equivalently the log returns are normally distributed. The GBM for the price at time \( t \) of a futures contract with maturity \( T \), denoted \( F_{t,T} \), is the martingale process, under the risk neutral probability measure \( Q \):

\[
dF_{t,T} = \sigma F_{t,T} dW_t
\]

where the volatility $\sigma$ is constant and $W$ is a Wiener process. Application of Ito’s lemma to the no-arbitrage relationship between spot and futures prices provides the following representation for the spot price $S_t$: \[ dS_t = (r - y)S_t dt + \sigma S_t dW_t \] (24.2)

where $r$ is the carry cost (including financing, transportation, storage, insurance, etc.) and $y$ is the convenience yield.

It is important to note that equations (24.1) and (24.2) will only be equivalent processes for the market prices of the spot and futures if the futures is fairly priced; that is, $F_{t,T} = F_{t,T}$. But commodity futures can fall far below their fair price relative to the spot market price because only a one-way arbitrage is possible (spot commodities cannot be sold short). The deviation of the market price of the futures from its fair price relative to the spot is attributed to the convenience yield, and this is very uncertain.

Therefore, the equations (24.1) and (24.2) are generally driven by different Wiener processes because the uncertainty in the basis appears the spot price, but this never changes the fact that the futures price must be a martingale under the risk-neutral measure.

**Spot Price Processes**

Spot prices can exhibit mean-reversion and seasonality, and their uncertainty includes uncertainty about the basis. This section explains how to extend the process (24.2) to allow for these.

Gibson and Schwartz\(^3\) introduced the following two-factor process with stochastic convenience yield:

\[
\begin{align*}
    dS_t &= (r - y)S_t dt + \sigma_1 S_t dW_{1,t} \\
    dy &= (\kappa(a - y) - \lambda) dt + \sigma_2 dW_{2,t} \quad \text{and} \quad (dW_{1,t}, dW_{2,t}) = \rho dt
\end{align*}
\] (24.3)

where $\kappa$ is the rate of mean reversion for the convenience yield, $\alpha$ is the mean convenience yield, and $\lambda$ is the convenience yield risk premium.\(^4\)

\(^2\)The No-Arbitrage Condition for the Fair Price $F_{t,T}$ of the Future is $F_{t,T} = S_t e^{(r-y)(T-t)}$.


\(^4\)This allows for risk adjusted drift as convenience yield risk cannot be completely hedged.
The fair value relationship between spot and futures under these processes is given by

\[ F_{t,T} = S_t \exp \left( -\gamma \frac{1 - e^{\kappa (T-t)}}{\kappa} + A_{t,T} \right) \] (24.4)

where

\[ A_{t,T} = \left( r - \alpha + \frac{\lambda}{\kappa} + \frac{1}{2} \frac{\sigma^2_2}{\kappa^2} - \frac{\sigma_1 \sigma_2 \rho}{\kappa} \right) (T - t) + \frac{1}{4} \sigma_2^2 \frac{1 - e^{-2\kappa (T-t)}}{\kappa^3} \]

\[ + \left( \left( \alpha - \frac{\lambda}{\kappa} \right) \kappa + \sigma_1 \sigma_2 \rho - \frac{\sigma_2^2}{\kappa^2} \right) \frac{1 - e^{-\kappa (T-t)}}{\kappa^2} \]

An alternative to modeling mean reversion in a stochastic convenience yield is to apply a mean reverting stochastic process to the spot price itself, such as the one-factor Pilipovic model:6

\[ dS_t = \kappa (X - S_t) dt + \sigma S_t^\gamma dZ_t \] (24.5)

where \( \kappa \) is the speed of mean reversion, \( X \) is the equilibrium price, and \( \gamma \) any positive real number. Beyond this we have multifactor mean reverting models that assume a stochastic, mean-reverting equilibrium price.7

These models are useful for pricing path dependent options on spot prices where the processes display mean-reversion linked, for example, to seasonal patterns.

### Jump Diffusion

One of the main limitations of GBM is that the underlying price has a lognormal distribution, yet this is rarely borne out in practice. Traders

---


know that asset returns have skewed and heavy-tailed distributions and this is the reason why we observe a volatility smile and skew in the market prices of plain European options. Heavy tails are a common feature in commodity returns distributions, particularly in energy and power markets where price spikes and jumps are frequent.\(^8\)

To capture such behavior a *jump diffusion* (JD) process is necessary. Taking the martingale process in equation (24.1) and adding a Poisson distributed random jump variable gives

\[
dF_{t,T} = F_{t,T}(-\lambda k dt + \sigma dW_t + Y_t dq_t)
\]

where \( q_t \) is a Poisson process, \( \lambda \) is the jump risk premium, \( k \) is the jump intensity, and \( Y_t \) is the magnitude of the jump, being a random variable with some specific distribution. A popular choice is to assume that \( Y_t \) is lognormally distributed, following Merton,\(^9\) because it gives an analytic formula for the option price. But lognormality implies that price jumps can only be positive, which is not a suitable assumption for energy and power markets where prices spike and can jump down as well as up. For such markets, double-jump processes are more realistic.

Jump diffusion models have theoretical and practical disadvantages. The inability to hedge all sources of risks means that we have an incomplete markets setting, and calibration of these models is very difficult.

**Stochastic Volatility**

In a general stochastic volatility framework, the underlying price and its variance follow correlated processes:

\[
\begin{align*}
    dF_{t,T} &= \sqrt{V_t}F_{t,T}dW_{1,t} \\
    dV_t &= \alpha dt + \beta V_t^\gamma dW_{2,t}
\end{align*}
\]

with \(<dW_1, dW_2> = \rho dt\)

The parameters \( \alpha \) and \( \beta \) can depend on \( F_{t,T} \) and \( V_t \), but then numerical methods must be used to obtain the price of even standard European options.

---

\(^8\) Jumps can occur in futures prices as well as spot prices so in the following, since most commodity options are on futures, we describe the futures price process leaving readers to infer the associated spot price process themselves.

One of the few stochastic volatility models with a (quasi) analytic solution for standard European options is Heston’s model.\(^{10}\) In this model \(\alpha\) is a mean reversion term with a volatility risk premium in the mean reversion rate, \(\beta\) is constant, and \(\gamma = 0.5\). Its popularity rests on the fact that it is relatively easy to calibrate.\(^{11}\)

The Heston model also has nonzero price-volatility correlation and this is essential if the model is to capture the skewed and leptokurtic price densities of commodity futures. With a zero correlation between the price and volatility, as for instance in the Hull and White model,\(^{12}\) the price density is leptokurtic but not skewed. The model-implied volatilities therefore must have symmetric smiles. This is unrealistic for almost all markets.

Recent additions to the family of stochastic volatility models are the stochastic-implied volatility model of Ledoit and Santa-Clara and Schonbucher\(^{13}\) and the stochastic local volatility model of Alexander and Nogueira.\(^{14}\) Stochastic implied volatility assumes a different, correlated stochastic process for each implied volatility and stochastic local volatility assumes the parameters of a deterministic volatility function are stochastic. Alexander and Nogueira prove that the two approaches are equivalent. They give identical option prices and hedge ratios, but stochastic local volatility models are easier to calibrate.

**Local Volatility**

The concept of local volatility was first introduced by Dupire\(^{15}\) and Derman.\(^{16}\) Local volatility \(\sigma(F_{t,T},t)\), also known as forward volatility, is the future volatility locked in by the prices of traded options, just as forward


interest rates are locked in by the prices of traded bonds. Since volatility is deterministic, markets are arbitrage free and we can find a unique local volatility surface that is consistent with any implied volatility surface.

Local volatility is a way to avoid complete specification of the price process and preserve the simplicity of Black-Scholes framework. There is only one source of risk, the markets are complete, and preference free option valuation is possible. Dupire derived a celebrated equation for the local volatility function:

\[
\sigma(F_{t,T}, t)| (F_{t,T} = K, t = T) = 2 \frac{\frac{\partial f_{K,T}}{\partial T}}{K^2 \frac{\partial^2 f_{K,T}}{\partial K^2}} \tag{24.8}
\]

where \( f_{K,T} \) is the market price of an option with strike \( K \) and maturity \( T \).

Local volatility implies that the martingale process for the futures price has nonconstant volatility. The local volatility \( \sigma(F_{t,T}, t) \) is a deterministic function of the underlying price and time. The difficulty lies in extracting a local volatility function from the market data that is stable over time. For this reason many practitioners now use the term local volatility to refer to any processes with a nonconstant but deterministic volatility. Many different parametric forms have been proposed, amongst the most popular being the lognormal mixture diffusion of Brigo and Mercurio\(^{17}\) where the price density is assumed to be a mixture of two or more lognormal densities. The lognormal mixture approach has two great advantages: It captures the skewness and leptokurtosis observed in price densities and it retains the tractability of lognormal models. In particular, the price of a European option is just a weighted sum of Black-Scholes option prices based on different volatilities.

**GARCH**

Market prices of options are always not easy to find. For instance there are no exchange traded options on electric power. Hence to price an OTC contract for an option on power futures, or for any other options where market data are not available, we may consider calibrating the option pricing model using historical data and adjusting the drift for risk neutrality.

It is possible to formulate discrete time versions of any of the continuous time processes described above and many of these will be equivalent to

a GARCH process. GARCH—for generalized autoregressive conditional heteroscedasticity—is the standard framework for modeling time varying volatility in discrete time and was introduced by Engle\textsuperscript{18} and Bollerslev.\textsuperscript{19} By now there are numerous different GARCH models and a vast literature on the comparative quality of their fit to historical data on returns. A survey of this is provided by Alexander and Lazar\textsuperscript{20} who demonstrate the advantages of using a GARCH model where the conditional returns distribution is a mixture of two normal distributions.

From this vast literature the consensus option is that an asymmetric GARCH model with any skewed and leptokurtic conditional returns distribution fits most financial returns far better than the plain vanilla symmetric normal GARCH (1,1) model:

\[ \sigma^2_t = \omega + \alpha \epsilon^2_{t-1} + \beta \sigma^2_{t-1} \]  

(24.9)

where $\omega > 0$ is a constant, $\alpha \geq 0$ is the error coefficient, and $\beta \geq 0$ lag coefficient.

It can be proved that the continuous limit of these models is a continuous time stochastic volatility model. Therefore, estimating GARCH model parameters using a series of historical returns allows one to infer option prices in a stochastic volatility framework. Nelson\textsuperscript{21} proved that the standard normal GARCH (1,1) model converges to a stochastic volatility model with zero price-volatility correlation. This is unfortunate since such models are of limited use. However, the assumptions made by Nelson were questioned by Corradi,\textsuperscript{22} and later work by Alexander and Lazar\textsuperscript{23} has not only shown that Nelson’s conclusion should be questioned, but that an assumption-free

\textsuperscript{23}Alexander and Lazar, “On The Continuous Limit of GARCH.”
continuous limit of (weak) GARCH is actually a wonderful stochastic volatility model! It takes the form:

\[
\begin{align*}
    dF_{t,T} &= \sqrt{V_{t,T}} dW_{1,t} \\
    dV_t &= (\omega - \theta V_t) dt + \sqrt{\eta - 1} \alpha V_t dW_{2,t} \\
    \langle dW_{1,t}, dW_{2,t} \rangle &= \rho dt \\
    \rho &= \frac{\tau}{\sqrt{\eta - 1}}
\end{align*}
\]  

(24.10)

The nonzero correlation \( \rho \) between the price process and the volatility captures a proper volatility skew, and the correlation is related to the skewness \( \tau \) and kurtosis \( \eta \) of returns, which is very intuitive.

**Forward Curve Models**

The single-factor models of futures prices that we have considered so far ignore any relationship between futures of different maturities. Yet term structures of commodity futures are very highly correlated and options that depend on more than one futures price, such as the calendar spread energy options that are actively traded on NYMEX, need to account for this correlation. The general forward curve model for commodities is similar to the HJM model for interest rates:\(^{24}\)

\[
\begin{align*}
    dF_{t,T} &= \sum_{i=1}^{m} \sigma_i(t, F_{t,T}) F_{t,T} dZ_{i,t} 
\end{align*}
\]  

(24.11)

where \( m \) is the number of uncorrelated common factors. These models are difficult to calibrate due to the large number of parameters and prices are often computed using Monte Carlo simulation.\(^{25}\)

**PRICING OPTIONS**

In this section we describe some common types of commodity options and, where possible, state their prices under different assumptions about the stochastic process governing the underlying price dynamics.

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Standard European Options

Under the assumption that the futures price follows the zero-drift geometric Brownian motion in equation (24.1), Black and Scholes\(^{26}\) derived the following analytic formula for the price at time \(t\) of a standard European option on \(F_t, T\) with strike \(K\) and maturity \(T\):

\[ f_{t}^{K,T} = \omega e^{-r(T-t)} (F_t \Phi(\omega d_{1,t}) - K \Phi(\omega d_{2,t})) \] (24.12)

where \(\Phi\) is the standard normal distribution function, \(\omega = 1\) for a call and \(\omega = -1\) for a put and

\[ d_{1,t} = \frac{\ln(F_t / K)}{\sigma \sqrt{T-t}} + \frac{1}{2} \sigma \sqrt{T-t} \] (24.13)

\[ d_{2,t} = d_{1,t} - \sigma \sqrt{T-t} \]

The associated formula for a European option on the spot price with GBM dynamics, equation (24.2) is the celebrated Black-Scholes formula:

\[ f_{t}^{K,T} = \omega (S_t e^{-\gamma(T-t)} \Phi(\omega d_{1,t}) - Ke^{-r(T-t)} \Phi(\omega d_{2,t})) \] (24.14)

Under the lognormal jump diffusion model of Merton\(^{27}\) the price of a standard European option is a Poisson distributed sum of Black or Black-Scholes prices with adjusted drift and volatility to compensate for the effect of the jumps. Specifically, in equation (24.7), suppose \(\log(Y_t)\) has a normal distribution with mean \(\alpha\) and standard deviation \(\beta\), that is, \(\log(Y_t) \sim N(\alpha, \beta)\). Then the price of a standard European option under the jump diffusion process is

\[ f_{t}^{K,T} = \sum_{n=0}^{\infty} \frac{e^{-\lambda \Delta t} (\lambda \Delta t)^n}{n!} f_{t}^{BS} \left(S, K, T, r - Ak + \frac{n\lambda}{T}, \sqrt{\sigma^2 + \frac{n\beta^2}{T}}, \omega\right) \] (24.15)

where \(A = \lambda, \omega = 1\) for calls, \(A = \alpha, \omega = -1\) for puts, and \(f_{t}^{BS}(S, K, T, r, \sigma, \omega)\) is the Black-Scholes price as in equation (24.14).


\(^{27}\)Merton, *Theory of Rational Option Pricing*. 
American Options

Before expiry, the possibility of early exercise means that the price of an American option is always greater than or equal to the price of its European counterpart. Since no traded options are perpetual the expiry date forces the price of an American option to converge to the European price.

The majority of exchange-traded commodity options are standard American options on futures. For a standard American call or put on a futures contract, and under the assumption that the premium is paid at expiry, it can be shown that the early exercise premium will not affect the price of the option. But of course option premiums are payable up front, so this theoretical result does not hold exactly in practice. The possibility of early exercise implies standard American options on futures may have prices above those of the corresponding European option, but the effect is quite small.

More generally, and for path dependent options such as the Asian options we discuss next, the price of an American-style option is determined by the type of the underlying asset, the prevailing discount rate, and if the option is on the spot price, also the convenience yield.

American options can be priced using the free boundary pricing methods of McKean, Kim, Carr et al., Jacka, and others. For instance, the price of a standard American option with payoff \( \max \{ \omega (S_t - K), 0 \} \) on a commodity with spot price process (24.2) is given by

\[
P(S_t, t, \omega) = P^E(S_t, T, \omega) + \omega \int_t^T yS_te^{-y(s-t)} \Phi(\omega d_1(S_t, B_t, s-t)) \, ds \\
- \omega \int_t^T rKe^{-r(s-t)} \Phi(\omega d_2(S_t, B_t, s-t)) \, ds \tag{24.16}
\]

where \( \omega = 1 \) for a call and \( -1 \) for a put and \( B_t \) is the early exercise boundary. That is, an American call option price is the price of its European

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28See James, *Option Theory*.


counterpart plus the income from dividends (after exercise) minus the risk-
free interest lost due to the payment of the strike price. At the boundary
(optimal exercise), the price of the American option is its intrinsic value;
that is, \( v(S_t - K) \), and the slope of the price function is one. These are called
value-match and high-contact conditions respectively. \( B_t \) is often estimated
numerically using a gradient algorithm.33

**Asian Options**

An Asian option reduces the risk faced by the writer and allows the holder
to secure his supplies at a cheaper price at the same time. For commodities
that are prone to frequent spikes or jumps, Asian options considerably
reduce the calendar basis risk. As the volatility of the average price is less
than the price itself these options are cheaper than their standard
counterparts.

There are two types of Asian options: average price options and average
strike options. The payoff to these is given by

\[
V_{\text{Average Price}} = \max\left(\bar{S}_{t_0,t_n} - K, 0\right)
\]

\[
V_{\text{Average Strike}} = \max\left(\bar{S}_t - \bar{S}_{t_0,t_n}, 0\right)
\]

(24.17)

where

\[
\bar{S}_{t_0,t_n} = \frac{\sum_{t_i=t_0}^{t_n} S_{t_i}}{t_n - t_0}, \quad 0 \leq t_0 < T; \quad t_n = T
\]

The averaging period can start right on day zero or at a forward date.
Contracts which involve trades with different volumes over a period of time
might use (volume) weighted averages.

Asian options are widely traded OTC and in recent years options on
futures have been introduced in exchanges worldwide. Exhibit 24.2
shows the dramatic increase in the volumes of Asian options, particularly
crude oil, traded in the last two years. Exchange-traded contracts are pri-
marily financially settled while an OTC contract might involve physical
delivery.

The most widely used techniques to price Asian options assume a pro-
cess of the form (24.2). But pricing under this assumption is not easy as the

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33For example, see Giovanni Barone-Adesi and Robert E. Whaley, “Efficient Ana-
lytic Approximation of American Option Values,” *Journal of Finance* 42, no. 2
average of the prices is not lognormal. There is no closed form solution and
the prices are often computed numerically or using analytic approxima-
tions. Few methods assume that average price is lognormally distributed
but the results are not accurate.34

An approximation by Vorst35 uses the difference between the arithmetic
and geometric averages to compute the price of the option. The advan-
tage of using geometric averages is the fact that a product of lognormal
variables remains lognormal. For example, for an Asian option on the spot
we have,

\[ f_t^G \leq f_t^A \leq f_t^G + e^{-r(T-t)}(E[S_A] - E[S_G]) \]  

(24.18)

The approximate price is given by

\[ \hat{f}_t = \omega e^{-r(T-t)}(S^* \Phi(\omega d_1^*) - K^* \Phi(\omega d_2^*)) \]  

(24.19)

\[
S^* = E[S_G] = e^{\mu G + \frac{1}{2} \sigma_G^2};
\]

\[
K^* = K - (E[S_A] - E[S_G]);
\]

\[
d_1^* = \frac{\ln(S/G) + \frac{1}{2} \sigma_G^2}{\sigma_G} \]

\[
d_2^* = d_1^* - \sigma_G
\]

where \(\ln(S_G) \sim N(\mu_G, \sigma_G^2)\).36

**Spread Options**

A standard spread option is just like a plain vanilla option but it is written on
the spread between two futures prices (or, less commonly, on the spread

35Ton Vorst, Prices and Hedge Ratios of Average Exchange Rate Options White
Paper, Econometric Institute, Erasmus University Rotterdam, 1990.
36For a similar (and better) analytic approximation see Michael Curran, “Beyond
Average Intelligence,” *Risk* 5, no. 10 (1992), p. 60.
between two spot prices). Spread options comprise a diverse range of products that are used to hedge a variety of risks, correlation and lock in revenues. A few examples are options on intercommodity spreads (cracks and sparks), intracommodity spreads (quality), calendar spreads, and locational spreads.

The most basic approach to pricing a spread option would be to assume the spread follows an arithmetic Brownian motion. But this ignores the correlation between the two price processes and would lead to inaccurate results. Ravindran Transpose, Shimko, Kirk and others assume the two prices follow correlated geometric Brownian motions (2GBM). Pricing European spread options in this framework is difficult because a linear combination of lognormal processes is not lognormal.

The analytic approximation to the price of a European spread option on futures was given by Kirk.

\[ P_t = \omega e^{-r(T-t)} \left( F_{1,t}\Phi(\omega d_1^T) - (K + F_{2,t})\Phi(\omega d_2^T) \right) \]  

The problem with approximations such as Kirk’s is that it is only valid for spread options with very low strikes. As soon as the option strike rises even to the at-the-money (ATM) level, the approximation is inaccurate. A much better approximation to the price of a spread option, one that is accurate for all strikes, has been developed by Alexander and Venkatramanan. They represent the price of spread option as the sum of prices of two compound exchange options and then apply the exchange option price derived by Margrabe. The exchange options are: to exchange a call on one asset with a call on the other asset, and to exchange a put on one asset with a put on the other asset. The risk neutral price of the spread option at time \( t \) is given by

\[ f_t = e^{-r(T-t)} \mathbb{E}_Q \left[ \left[ \omega (U_{1,T} - U_{2,T}) \right]^+ \right] + e^{-r(T-t)} \mathbb{E}_Q \left[ \left[ \omega (V_{2,T} - V_{1,T}) \right]^+ \right] \]  

References:
where $U_{1,T}, V_{1,T}$ are payoffs to European call and put options on asset 1 with strike $mK$ and $U_{2,T}, V_{2,T}$ on asset 2 with strike $(m - 1)K$, respectively. $E_Q$ is the expectation under the risk neutral measure and $\omega = 1$ for calls, $-1$ for puts.

Because the payoff to a spread option decreases with correlation, “frowns” in the correlation implied from market prices of spread options of different strikes are evident. Market prices of out-of-the-money (OTM) call and put spread options are higher than the standard 2GBM model prices based on the ATM implied correlation, because traders recognize the skewed and leptokurtic nature of commodity price returns. Hence the implied correlations that are backed-out from OTM options in the 2GBM model are lower than the ATM implied correlation.

A model that captures this feature is the stochastic volatility jump diffusion of Carmona and Durrleman. However, pricing and hedging in this model necessitates a computationally intensive numerical resolution method such as a fast Fourier transforms. An alternative is to use the bivariate normal mixture approach of Alexander and Scourse, which provides an analytic approximation to the price of a spread option that is accurate, consistent with implied volatility skews, and also consistent with correlation frowns in spread option market prices.

**SWING OPTIONS**

Swing options are volumetric contracts that are mainly traded in markets which require a high degree of flexibility in the delivery of the physical asset. For instance, in natural gas markets, since storage capacities are limited, the distributor might require variable supplies due to sudden changes in demand from the end user. In a typical contract, the holder of the option agrees to buy a fixed amount of gas (base amount) and has an option to raise or decrease his required quantity (swing) within a prespecified limit for the agreed strike price.

A swing contract with $N$ days to expiry would allow the holder to exercise $n \leq N$ swings at a rate of one per day. When $n = N$ the pricing problem reduces to pricing a strip of $n$ European options with corresponding strikes and maturities. When $n < N$ then the problem becomes that of optimal exercises equivalent to pricing $n$ early exercise options. When $n = 1$ then the

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price is that of a single American option. This gives us a range of prices between which the swing option price must lie:

\[ p_{\text{European}}^{n=N} \leq p_{\text{Swing}}^{0 < n \leq N} \leq p_{\text{American}}^{n=1} \]

Swing options can be priced dynamically using \( K \) simultaneous 2-D trees in a similar fashion as the American options.\(^{44}\)

**SUMMARY**

Commodity options are traded for portfolio diversification, speculative, and risk management purposes. Most of the activity is on the U.S. exchanges where options on energy futures, metals futures, and agricultural futures are traded. The majority of these options are standard American calls and puts; but the market for calendar spreads and average price options has been growing during the last few years.

The historical characteristics of commodity prices are specific to the commodity type. We have examined five representative commodities:

- **Corn.** Where the market is now usually contango and price jumps are associated with news.
- **Live hogs.** Where futures prices are not highly correlated and seasonal price peaks occur in summer months.
- **Silver.** Where the term structure is almost flat, there is no seasonality and prices jump with speculative trading.
- **Natural gas.** Where the term structure swings between backwardation in winter and contango in summer, and prices can spike up during winter cold snaps and down in the summer when storage is full to capacity.
- **Electricity.** Where spot prices are excessively volatile in the summer, futures prices are highest in the winter and the term structure has jump for futures expiring in winter and spring.

Almost all commodity options prices can be based on a martingale process for the futures, possibly with jumps. The only exception is path

dependent options on the spot price, for which a spot price process with mean reversion, jumps, and possibly a stochastic convenience yield could be used.

American options on futures have prices that are either equal to or very close to those of the equivalent European options when the premium from the discount rate is very small. Analytic formulas or approximations have been given for standard options, average price options, and spread options and these are the options that are most actively traded.