This chapter introduces Bayesian belief and decision networks as quantitative management tools for operational risks. Bayesian networks are already well-established for use in the risk management of large corporations and the aim of this chapter is to describe how these powerful statistical tools may be applied to operational risk management in banks and other financial institutions.

In order to manage operational risks effectively, the factors that are thought to influence the risk must be identified. These can be the “key risk drivers” of the firm’s operations (see sections 12.4.4. and 13.7) or they can be classified into a separate category of their own (see section 12.2). Ideally, by exerting some control over these factors, operational risks will be reduced. Before attempting this, some important questions should be addressed:

1. **Effectiveness**: What effect will the controls have on the risk? It is one thing to identify the factors that contribute to the risk, but quite another to actually quantify the effect that changes in a factor will have upon the risk. For this one needs a quantitative model that relates the risk to the factors. This is precisely what a Bayesian network is.

2. **Dependency**: Is it possible that by reducing one risk, another risk will be increased? How can one quantify the dependency between risks, and can this dependency also be controlled? Managing the dependency between risks is one of the main strengths of a Bayesian network. In scenario analysis, where many possible scenarios are examined by risk control, the risk dependency structure is explored in a very systematic way. An example of this will be given in section 14.3.

3. **Cost**: What will be the cost of the controls, and is the likely reduction in risk worth the expense? The answer to this question will depend on the risk attitude of the firm – and this risk attitude is usually modelled by a utility function over costs and benefits of a decision (see the appendix to Chapter 15). In section 14.4 we shall explain how to incorporate a utility function into the management decision process, using an influence diagram called a Bayesian “decision” network.

The outline of this chapter is as follows: The first section provides references and website links for further information on Bayesian networks and their applications. Section 14.2 introduces the reader to Bayesian networks: their architecture (parent and child nodes, initial and terminal nodes, the edges that imply risk attribution and so forth) and their propagation – that is, how the parameters are estimated using Bayes’ rule. Section 14.3 gives an example of the design of a Bayesian network for the management of operational risks in banks. It highlights the major advantage of Bayesian networks, which is the scenario analysis of operational risks. Section 14.4 explains how Bayesian networks can be augmented to include decision nodes, and thereby facilitate a cost-benefit analysis of a management decision. Bayesian decision networks enable management decisions to be more informed because the choice of management action can be evaluated in terms of different scenarios on the important attributes of the operational risk. Section 14.5 concludes.
14.1 Bayesian Networks: Useful References and Web Links

The extensive literature on Bayesian networks goes back over a decade: see Pearl (1988), Neapolitan (1990) and Jensen (1996). For many years Bayesian belief and decision networks have been used very successfully in the management and decision sciences. See Geoffrian (1987), Morgan and Henrion (1990), Heckeman et. al. (1995) and Henrion et. al. (1986). Other important applications of Bayesian networks and influence diagrams include reliability analysis - see Fenton and Littlewood (1991) and the design of expert systems - see Henrion et.al. (1991) and Neapolitan (1990). Applications of Bayesian networks to modelling operational risks in banking and finance have been described in Alexander (2000, 2001) and King (2001).

Characteristically, one of the first risk management software vendors in the financial industry to offer a Bayesian network product was Algorithmics: see www.algorithmics.com. For an example of commercial software that uses Bayesian networks to manage operational risk in corporates, see www.lumina.com. There are several software packages for Bayesian networks that are freely downloadable from the Internet. The examples in this chapter have been generated using an excellent package called 'Hugin lite' (downloadable free for research purposes from www.hugin.com or www.hugin.dk). Microsoft provides a free package for personal research only that is Excel compatible at www.research.microsoft.com/research.dtg/msbn/default.htm and a list of free (and other) Bayesian network software on the web is on http.cs.berkeley.edu/~murphyk/Bayes/bnsoft.html

An interesting new initiative from www.inferspace.com is to provide free, open-source software for Bayesian networks. Intel have initiated the "OpenBayes" system, a translation of the Bayes Net Toolbox (BNT) Matlab package. The APIs are written in C++, but functions can be written to enable the functionality of the APIs to be accessed from within mathematical and statistical software packages, such as S-Plus and Mathematica.

14.2 Introducing Bayesian Networks

A Bayesian network is a statistical model that relates the marginal distributions of “causal” factors, or “attributes” of a risk, to its multivariate distribution. The basic structure or “architecture” of a Bayesian network is a directed acyclic graph where nodes represent random variables and links represent relationships between the variables. Figure 14.1 illustrates on the right the architecture of a simple Bayesian network with a single target or “terminal” node Z having two “parent” nodes X and Y; on the left the distributions associated with each node are displayed. The initial nodes are nodes with no parents (i.e. X and Y in this example). They each have univariate distributions that must be specified by the modeler: in figure 14.1 the probability that X is in state 0 is 20%, and so forth. The terminal node Z has a multivariate distribution that is determined by the initial node’s distributions and conditional distributions (the probability that Z is in state “a” given that X is in state 2 and Y is in state 1, and so forth). These conditional probabilities are not shown in figure 14.1 and only the joint distribution of the target node is shown in the left hand frame.
A network uses Bayes’ rule to “propagate” through the network, and thus the distributions at all nodes can be quantified, given the initial node probabilities and the conditional probabilities for all nodes. For two events Y and Z, Bayes rule is:

$$P(Z \mid Y) = P(Y \mid Z) P(Z) / P(Y)$$

Moreover, if the states of any nodes are fixed, the network can use Bayes’ rule to propagate backwards through the network and hence calculate the posterior probabilities of every node in the network. This is the basis of scenario analysis in Bayesian networks, and it is one of the most attractive features of Bayesian networks. The ability to perform scenario analysis in this rigorous, but also tractable and visual manner should be viewed as the over-riding reason for their use.

Figure 14.2 illustrates how a scenario analysis is implemented in the simple Bayesian network of figure 14.1. The initial state of the network is as shown in figure 14.1. But suppose that we design a risk control so that X will be in state 0 – what then would be the distribution of Z? Alternatively we might ask, what are the conditions that lead to a given state of the terminal node: for example, suppose we observe that Z is in state “a”, then what are the posterior probabilities associated with X and Y, given this information? Questions like this, which form the basis of an operational risk scenario analysis, are very easy to answer with a Bayesian network. The network propagates, using Bayes rule, to calculate the posterior probabilities of each node, given the information in the scenario.

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1 See Bernardo and Smith (1994)
Figure 14.2 shows the results of applying Bayes rule to the two scenarios that have just been described. Given the risk control, our prior belief is that X will be in state 0, and then the posterior density of Z is shown below in the left hand frame; the probability that it will be in state “b” is reduced, but the probabilities that X will be in state “a” or state “c” have increased. We may then ask whether the control is likely to produce the desired results. For the second scenario, given the observation that Z is in state c, the posterior probabilities of X and Y are shown above it, in the right hand frame. For example, with no extra information the probability that Y is in state 0 is 30%. But if we have observed that Z is in state “a”, then we know that Y is far more likely to be in state 0, in fact the probability of Y being in state 0 rises to 56%.

In figures 14.1 and 14.2, very simple discrete random variables have been used, but a Bayesian network has the flexibility to model discrete random variables with many states, and discrete or continuous random variables from some family of distributions where parameter values themselves have distributions that are conditional on the states of the parent nodes. We shall see an example of this in the next section, when scenario analysis in a Bayesian network that targets a “Key Risk Indicator” (KRI) of an operational loss will be developed.

14.3 Applications of Bayesian Networks in Banking and Finance

There is no unique Bayesian network to represent any situation, unless it is extremely simple. Rather, a Bayesian network should be regarded as the analyst's own particular view of a process. Many different Bayesian networks could be used to depict the same process.¹

When designing Bayesian networks for scenario analysis of operational risks in bank, terminal nodes can be the key risk indicators that have been identified, and agreed upon, as targets for control. Examples of key risk indicators include the number of failed trades, staff turnover rates, or the frequency and/or severity of errors. The “causal” factors or “attributes” of the risk will be identified with “key risk drivers” such as volume of transactions processed, product complexity, and systems downtime. See Chapters 12 and 13 for further discussion of risk drivers and risk indicators.

For a fully integrated view of management and capital allocation, a Bayesian network could have terminal nodes corresponding to the number of loss events and the loss given event. Thus the Bayesian network will model the frequency and severity distributions, and therefore their composite (the annual loss distribution) as functions of the key risk drivers in the firm.

In this way, the management and control of operational risks can be linked to the economic capital of a firm, or the regulatory capital of a bank. Furthermore, the Bayesian network will allow management decisions to be supported by scenario analysis, and to be integrated with the risk capital and budgeting of the firm.

¹ If a node has many parent nodes these conditional probabilities can be difficult to determine because they correspond to high dimensional multi-variate distributions. An alternative approach is to define a Bayesian network so that every node has no more than two parent nodes. In this way the conditional probabilities correspond to bivariate distributions, which are easier for the analyst to visualize.
Bayesian networks can also be used to determine the “trigger levels” associated with a key risk indicator. The trigger levels are bounds that determine various actions, that must be taken by management, if the risk indicator crosses that level. All nodes in a Bayesian network (with more than one state) are random variables. So when a key risk indicator is used as a target node in a Bayesian network, the network will determines its distribution, under any given scenario. This includes the mean, the standard deviation and the upper percentiles of the key risk indicator. From the initial state of the network, trigger levels can be set at either some multiples of the standard deviation or, if the distribution is skewed or fat-tailed, the upper percentiles. The precise levels at which trigger level are set will, of course, depend on the risk aversion (the more risk averse, the lower the percentile). A variety of trigger levels may be set, for example at increasing percentiles, and the trigger levels at higher percentiles should prompt more drastic actions than those at lower percentiles.

Having determined the trigger levels, the Bayesian network can then be used to decide on the most appropriate risk controls when trigger level are exceeded. Through scenario analysis, the Bayesian network can answer questions such as:

- “Suppose the key risk indicator “staff turnover” enters the “black flag” zone. What then would be the probability that the pay scale is too low, and is this more likely to be a result of bad management, or poor training for our employees?”; or,

- “If the number of failed trades has entered the “red flag” zone will increasing staff levels be the best course of action that is most likely bring failures down to acceptable levels again?”

Clearly, answers to this type of question will help management to decide on the best course of action when trigger levels are exceeded. More details may be found in Alexander (2000, 2001). For reasons of space, only one Bayesian network for operational risk in banking will be presented in this chapter. The network shown in figure 14.3 represents the number of failed trades in, for example, the interest rate swaps desks of the bank.

The nodes in the network are as follows:

**Instrument**: The initial node “Instrument” represents all the over-the-counter (OTC) trades in interest swaps, including their hedging with other swaps and listed instruments such as futures and bonds. The probabilities of 0.5 on OTC and 0.5 on listed instruments, represents that 50% of the number of trades during a particular time interval (let us say one week) are in OTC swaps, and 50% of the number of trades in one week are in the listed instruments for hedging.

**Agreement**: Although the terms and conditions are likely to have been agreed before the swap, the master agreement may not have been finalized before the deal is made, and this is represented by the node “Agreement”. The node probabilities shown represent a 5% chance that the master agreement has not been finalized before the deal. This has been calculated (using Bayes rule) assuming that only 90% of the OTC trades have finalized master agreements, and recalling that 50% of the trades are OTC.

**Valuation and Booking**: The failure may occur on the internal side of the deal, over which the bank has some control, or on the external side. The number of fails on both sides are represented by continuous nodes that are conditional on both the valuation and the booking of the trade. The distributions of the internal and external valuation and booking nodes are given on the left hand side of figure 14.3.
Figure 14.3: A Bayesian Network for the Number of Failed Trades

Internal and External Number of Fails: The conditional probabilities that a deal fails, given that it is incorrectly or correctly booked, and incorrectly or correctly valued, and given that a legal master agreement has or has not been finalized will determine the distributions of these nodes. Figure 14.4 shows the conditional densities as those of normal variates, with a mean and variance representing the number of fails in a given time frame, for example one week. Thus the number of failed trades per week, arising on the external side, given that both valuation and booking are correct but when no legal master agreement has been finalized before the deal, is normally distributed with mean 50 and variance 50. The number of fails per week, on the external and internal side are therefore mixtures of normal densities, with the distributions shown in the monitor windows next to these nodes.

Figure 14.4: Conditional Distributions for Fails_EXT and Fails_INT
Number of Fails: The distribution of the target node, the number of failed trades per week, is assumed to be a mixture of these two densities, with probability 0.5 on each. That is, we assume that it is equally likely that a fail will arise from either side of the trade.

For reasons of space we do not list all the conditional probabilities of every node in the network: but below we shall consider the conditional distribution of the internal booking node, so we need to know that 85% of OTC trades and 90% of listed trades are correctly booked, internally.

The initial state of the network may be used to set trigger levels for the key risk indicator “number of fails”. In this case, with a mean number of failed trades per week of 30.78, with standard deviation 23.73, a trigger level might be set at [mean + 2 standard deviation] ≈ 80. Another trigger level, prompting more drastic action if it is exceeded, might be set at [mean + 3 standard deviation] ≈ 100 failed trades per week. If a risk indicator such as number of failed trades exceeds a trigger level, the management will be prompted into some sort of action. But which is the most efficient action to take (we shall leave aside the question of costs to the next section): for example, is it better to have a reviewing the internal booking procedure, or to ensure that no trades commence until the master agreement has been finalized. Which action will be most efficient in reducing the number of failed trades?

The Bayesian network will allow the manager to simulate what can happen if one of these controls is put in place. For example, figure 14.5 shows the effect on the number of failed trades if the internal booking of a deal is always correct. The network propagates forwards and backwards through each node, using Bayes’ rule and we shall now go through this in some detail:

Let us derive the posterior probabilities of the “Instrument” node, given that the internal booking is correct. Let Y be the event "the instrument is listed" and Z be the event "the internal booking is correct", so the posterior probability P(Z | Y) = 0.514286 in figure 14.5.

Without any information on the booking, in the initial state of the network we assumed that 50% of the instruments are listed and that 85% of OTC instruments and 90% of listed instruments are correctly booked internally. Now using Bayes’ rule we have the posterior probability that the instrument is listed, given that it has been correctly booked, as

\[
P(Y | Z) = \frac{P(Z | Y) \ P(Y)}{P(Z)} = \frac{0.9 \times 0.5}{0.9 \times 0.5 + 0.85 \times 0.5} = 0.514286
\]

In this scenario, the network shows that if the management were able to ensure correct internal booking, the mean number of fails should reduce from about 31 trades to about 26 trades per week, and the standard deviation would be reduced from about 24 to about 20. So, if the key risk indicator “number of fails” exceeds a trigger level, should management attempt to improve the booking procedures? What is the likely effect of other risk controls – are they more efficient? The Bayesian network can be used with other scenarios, in exactly the same manner as we have just illustrated, to evaluate the effect of possible risk controls on the distribution of the risk indicator.
14.4 Bayesian Decision Networks

So far, so good, but there is another important question, and that is whether it will be cost effective to implement a risk control. Having identified the most efficient risk control, through scenario analysis on the Bayesian network as described above, we now have to ask: will the cost of the control exceed the benefit? To answer this question, the Bayesian network must be augmented with decision and utility nodes, in which case it becomes a particular type of influence diagram called a Bayesian decision network.

The decision network in figure 14.6 models the probability of a failed trade due to internal valuation or booking errors, or a dispute arising with the counterparty when the master agreement had not been finalized before the trade. The pink decision node labeled “Control” represents three possible risk controls, each aiming to reduce the probability of a failed trade. This will be by reducing either the number of valuation errors, or the number of booking errors, or the probability that legal agreements have not been finalized before the trade.

In the initial state shown in figure 14.6, all three controls are in place, but although a risk control cost node “Cost1” is depicted in the network, for the initial state of the network no costs have been assumed. The other cost node “Cost2” represents the cost of a failed trade. For the initial state this is fixed at unity, and in this case the values in the left hand frame associated with the “Control” node are just proportional to the conditional probabilities of a failed trade, given that the fail occurred in booking, valuation and agreements, respectively. That is, there are three possible causes of a failed trade: with probability 11.1518% it will fail...
because it has been incorrectly booked, with probability 29.6765% it will fail because of incorrect valuation, and with probability 30.1217% it will fail because of a dispute over an un-finalized master agreement.

When costs are associated with the controls in “Cost 1”, and the cost of a failed trade is not unity, the values associated with the “Control” node represent the relative cost of implementing each control, assuming a linear utility function. For example, if a cost of 10 is associated with a failed trade and a cost of 20 is associated with each control, the costs associated with the “Control” node would be 20.1115, 20.2968 and 20.3012. The least cost control in this case is to reduce the number of errors in the booking process. On the other hand, if a cost of 1000 were associated with a failed trade and costs of 30, 5 and 10 are allocated to the booking, valuation, and agreement controls respectively, then the net costs are 41.15 for controls on the booking process, 34.68 for controls on the instrument valuation process, and 40.12 for controls on the finalization of the master agreement. The least cost control in this case would be to improve the valuation process.

**Figure 14.6: A Bayesian Decision Network**

This example has just illustrated the cost-benefit analysis of risk control using a linear utility function – more general utility functions that reflect risk the aversion of the firm may also be employed in the Bayesian network framework. More detail on utility functions and their crucial role in operational risk management are given in the next chapter.
Finally, the decision network may be used in scenario analysis, in just the same way as in section 14.3. With scenario analysis, management decisions can be based on a cost-benefit analysis of such questions as: “What is likely to be the most cost effective control for the risks of failed trades in the interest rate swaps desk?”.

14.5 Conclusion

When all is said and done, a Bayesian network is simply a model for a multivariate distribution. As with every model, it is not unique; it is a picture of the mind of the modeller. There is no universal Bayesian network that models an operational risk; the network must be specific, not only to the institution, but also to the management role.

Bayesian networks have been applied for many years in large corporations. Advantages of using Bayesian networks for operational risk management in banking and finance include:

- Bayesian networks have applications to a wide variety of operational risks. Conditional probabilities may be based on scorecard and/or historical data from the trading book or balance sheet. They can be used to model loss distributions, or the distributions of key risk indicators;

- A Bayesian belief network relates the factors that are thought to influence operational risk (the “key risk drivers”) to the risk measures or “key risk indicators” of the firm. This type of process model of risk can provide explicit incentives for behavioural modifications. Also, when a key risk indicator is the target node, the Bayesian network can be used to set the trigger levels and to evaluate the effectiveness of risk control.

- Bayesian decision networks provide a cost-benefit analysis of risk controls, where the optimal controls are determined within a scenario analysis framework.

This chapter has provided some examples where Bayesian networks are designed to answer specific questions, and in each case this is achieved through scenario analysis in the network. In my opinion, the ability to perform scenario analysis in a quantitatively rigorous, but also tractable and visually intuitive manner, is the over-riding reason for choosing Bayesian network modelling as the key operational risk management tool.
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