Binomial Gammas

*The Basle II gamma question is vexing minds around the world. Here concerned UK couple **Mr and Mrs J Pézier** of Purley, Surrey tell of their fears*

As a family we are very much concerned that global banking regulators may not have fully thought through their proposals for an internal measurement approach to calculating an operational risk capital charge.

Indeed, it is not clear to us how the so-called gamma factors to be used in calculating the charge under this approach will be assessed given that the exposure indicators used in the calculation may be based on the value of transactions or the value of assets without specifying the number of possible loss events.

This apparently arcane point is crucial.

From 2004, large international banks will for the first time have to set aside capital against possible losses from fraud, computer failure, trade settlement foul-ups and other operational hazards under Basle II, the new capital adequacy accord proposed by the Basle Committee of banking supervisors from the Group of 10 leading economies.

The European Union Commission intends imposing the Basle II operational-risk capital charge on all banks and investment firms within the 15-nation EU from the same date. Both the Basle Committee and the EU Commission will finalise their plans by the end of this year.

The internal measurement approach is the most complex of the three-stage approach to calculating operational risk capital under Basle II. The more refined the approach used by a bank, the lower the charge is expected to be.

In stage 1 (the basic indicator approach) operational risk capital will be a proportion of a simple volume indicator such as gross revenue; in stage 2 (the standardised rules approach) the charges will be disaggregated according to business lines; and in stage 3 (the internal risk-based approach) the charges for each business line will be internally calibrated according to the type of operational loss. Apparently there are discussions concerning the implementation of a fourth stage where banks would have more flexibility to base capital charges on internal models for operational loss distributions.

Stage 2 models require no internal loss event data; the main qualifying criteria are that independent operational risk management and control functions be established and that their business lines are properly mapped to the standardized lines of business. Stage 3 models will, however, require the bank to collect internal loss event data over several years, so we suppose that this effort will begin immediately for banks that aim to qualify for a stage 3 model when the new accord is implemented in 2004. As we understand the proposals, these data will be used to estimate the probability of an operational loss event (p) and the loss given the event (L) using an internal model that must be validated by the regulators. Of course, the loss given the event is generally uncertain; we shall denote its mean and variance by μ_L and σ_L^2.

Allow us to introduce another parameter not currently central to the Basle Committee proposals, that is, the total number of events (N) susceptible of operational losses during a pre-specified time interval. Basle II favours one year. For example, in retail and commercial banking, N would be the total number of a certain type of transactions processed during a year. In corporate finance, N would be the number of new deals that may lead to a write-down loss because of operational errors (for example, illegal or unauthorized).

In the first example one year of data may be sufficient to estimate the probability p and the average loss from failed transactions μ_L with a fair degree of precision, because N will be large and the standard deviation of the transaction processing losses σ_L may be relatively small compared to the average loss. However, in the corporate finance example, N will be relatively small and many years of data (if relevant) may be needed to estimate p and μ_L with a reasonable degree of precision; σ_L may be large.

To keep the model simple, let us assume independence between loss events. Then, the parameters N and p and the random variable L correspond to those of a binomial distribution B(N, p) on the states (0, L). That is, the total loss is the result of N independent 'Bernoulli' trials where in each trial the probability of losing an amount L is p; otherwise, with probability (1 − p), nothing is lost. In this model the expected total loss μ is simply calculated as:

$$\mu = N\, p\, \mu_L \qquad (1)$$

For example if 25,000 transactions are processed in a year by a back office, the probability of a failed transaction is 0.04 and the expected loss given that a transaction has failed is $1000, the expected total loss over a year is $1 million. On the other hand, if 50 investment banking deals have been done in one year, the probability of an unauthorized or illegal deal is 0.005 and the expected loss if a deal is unauthorized or illegal is $4 million, then the expected total loss will also be $1 million. However, as we shall see later, the distribution of the losses will be very different.

In stage 3 models the Basel committee proposes to estimate expected losses by using the formula:

$$\mu = E\, p\, L \qquad (2)$$

where E is an 'exposure indicator' that characterises the operational risk exposure in each line of business and risk type and L is the expected loss given a loss event,. The Basle Committee is considering how to set these exposure indicators. If E were to represent the *number* of transactions or deals, in other words if it were related to the number of events susceptible to loss, then the Basle Committee formula (2) would be match the binomial formula (1) where L is not regarded as a random variable but is equal to the average μ_L. We shall also be able to estimate the distribution of the losses, as described below.

However, as we understand the proposals, the *volume* (defined as total value) of transactions or trades, or the *value* of assets (in custody, under management or fixed) are currently being considered as possible exposure indicators. This raises questions about the estimation of a loss distribution. *The number of events is necessary to calculate the unexpected losses and, consequently, capital charges. We are, therefore, curious to examine the theoretical foundations on which capital charges will be justified by the Basle II proposals*.

The expected loss should be taken into account in valuing a business activity. Capital, on the other hand, is required as a buffer for unexpected losses, that is, to cover the 'tail' loss of a loss distribution. For market risk this is defined as the loss level that has not more than 1% chance of being exceeded during a two-week period and the capital buffer is set at three times this level. If the analytic form of the loss distribution is known, the percentiles can often be defined as a multiple of the standard deviation. For example, if the (profit and) loss distribution is normal $N(0,\sigma^2)$, the 1% tail loss level is at 2.33σ from the mean. A similar result would hold for any near normal distribution as would result from the sum of many losses. Let us then assume that capital requirements are set at $k\sigma$ and that k is approximately 7 for operational risks, as it is for market risks.

In stage 3 models for operational risks the Basle committee is proposing that the capital charge be a factor 'gamma' (γ) times the expected loss, that is, the committee assumes that unexpected loss is directly proportional to expected loss, where gamma is the proportionality constant:

$$\text{Capital charge} = k\,\sigma = \gamma\,\mu \qquad (3)$$

If the loss distribution is the binomial distribution B(N, p) on the states (0, L) then the assessment of gamma is simple: we know that unexpected loss can be assumed to be proportional to the standard deviation and, in the binomial model, the standard deviation and the expected loss have a simple relationship. For small p and known L, the total loss standard deviation is approximately $L\sqrt{(Np)}$ [1]and therefore

$$\text{Capital charge} = k\,L\,\sqrt{(Np)} \qquad (4)$$

Or, alternatively

$$\gamma = k\,/\,\sqrt{(Np)} \qquad (5)$$

Thus , in the simple binomial model, gamma and the capital charge are inversely proportional to the square root of the expected number of loss events, Np, during the pre-specified time interval.

Returning to our first illustration of a back office processing 25,000 transactions per year, each with a probability of a failure of 0.04 and an average cost per failure of $1000, the expected number of failures is 1000 and the total expected loss is $1 million. Assuming k = 7, by (5) gamma is $k/\sqrt{1000} \approx 7/31.6$ or 0.22 and by (3) the capital charge is only $220,000. In general, *for high frequency low impact operational losses, the gamma should be much less than 1. The capital charge should be much smaller than the expected loss and, therefore, more attention should be given to the estimation of predictable expected losses than to the determination of capital charges for unexpected losses*.

In the corporate finance example where only 50 deals are done per year and the probability of an operational failure per transaction is 0.005, the binomial gamma would be $k/\sqrt{0.25} = 2k = 14$, leading to a capital requirement of $14m ; this is 63 times larger than for the back office transactions processing example. Some of our assumptions such as 2.33 standard deviations to determine the 1% loss threshold are of course questionable in this case of low frequency events, but a direct calculation[2] shows that the 1% threshold will be between 2 and 3 loss events. Two loss events would lead to an expected loss of $8 million and therefore the $14 million capital charge is rather on the low side compared to the three times buffer size used

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[1] For a known loss, L, per loss event, the variance of total loss is $Np(1-p)L^2$, but since the probability of a loss event, p, is rather small, $1 - p \approx 1$. Therefore the total loss standard deviation is approximately $L\sqrt{(Np)}$.

[2] The probability of 3 or more loss events per year is down to one chance in a thousand whereas 2 losses or more has a probability of about 2.6% per year, that is, well above the 1% unexpected loss threshold.

for market risk. *For low frequency high impact operational losses, the gamma should be much larger than 1, the capital charge should be much greater than the expected loss and, therefore, attention should be given to predicting the severity of potential loss events and the probability that one or more could occur.*[3]

Indeed, for some type of operational losses, particularly the exceptional, low frequency losses, the severity of a loss may be highly uncertain. Taking into account the variance of the severity of a loss, σ_L^2 ,in addition to its expected severity, the total loss variance is approximately $Np(\mu_L^2 + \sigma_L^2)$. Thus (3) becomes:

$$\text{Capital charge} = k\sqrt{[Np(\mu_L^2 + \sigma_L^2)]} \quad (6)$$

Or, alternatively

$$\gamma = k\sqrt{[(1 + (\sigma_L/\mu_L)^2)/(Np)]} \quad (7)$$

Variability in the severity of loss given the event will increase gamma and therefore the capital charge. Returning to the corporate finance example, it may be reasonable to suppose that the operational loss will be highly variable; the standard deviation of the loss could be equal to its expected value. Consequently the gamma and the capital charge would increase by a factor of $\sqrt{(1 + (\sigma_L/\mu_L)^2} = \sqrt{2}$. That is, the gamma will increase from 14 to about 20 and the capital charge will reach $20 million. However for high frequency, low impact loss events, the uncertainty about the severity of each loss is likely to be much smaller (compared to the expected loss) and the effect of uncertainty in loss severity is unlikely to increase capital charges significantly.

The probability of a low frequency event will also be more difficult to estimate than the probability of a high frequency event. This has no impact on total expected loss, which depends only on the expected probability, but it will increase the size of the gamma for activities with low frequency operational risks.

In summary, high frequency low impact operational risks have relatively large p but relatively low μ_L and σ_L. Formulae (6) and (7) show that this type of risk will have a relatively low gamma and capital charge. However, low frequency high impact operational risks, which have very small and difficult to estimate probabilities and are associated with large and highly variable losses should have high gammas and, consequently, relatively high capital charges.

This short article has shown how total operational loss associated with a given risk type can be modeled with a binomial distribution B(N,p) on the states (0, L). In this model, the gammas and the capital charges for a stage 3 model are completely determined by the number of events N, the probability of a loss event p and the distribution of the loss given the event L. If the new Basle accord recommends the use of an exposure indicator related to the volume of trade or the value of assets under management without specifying the number of possible loss events, it is not clear to us how the gammas will be assessed.

Dr Jacques Pézier is Head of Marketing UK at Crédit Agricole Lazard Financial Products Bank, London: Tel: 44-(0)20-7815-1906 email Pezier@calfp.co.uk. Carol Alexander, his wife, is Professor of Risk Management at the ISMA Centre at Reading University: Tel: 44-(0)1189-316431 email c.alexander@ismacentre.rdg.ac.uk

[3] We have assumed that losses are independent, but there could be dependencies between low frequency loss events.