Does model fit matter for hedging? Evidence from FTSE 100 options

Carol Alexander **Andreas Kaeck**

ICMA Centre, Henley Business School, University of Reading, UK

June 4, 2010

ICMA Centre Discussion Papers in Finance DP2010-05

ICMA Centre • The University of Reading
Whiteknights • PO Box 242 • Reading RG6 6BA • UK
Tel: +44 (0)1183 788239 • Fax: +44 (0)1189 314741
Web: www.icmacentre.rdg.ac.uk
Director: Professor John Board, Chair in Finance
The ICMA Centre is supported by the International Capital Market Association

ABSTRACT

This paper implements a variety of different calibration methods applied to the Heston model and examines their effect on the performance of standard and minimum-variance hedging of vanilla options on the FTSE 100 index. Simple adjustments to the Black-Scholes-Merton model are used as a benchmark. Our empirical findings apply to delta, delta-gamma or delta-vega hedging and they are robust to varying the option maturities and moneyness, and to different market regimes. On the methodological side, an efficient technique for simultaneous calibration to option price and implied volatility index data is introduced.

JEL: G13, C13, C63

Keywords: Hedging, European Options, Stochastic Volatility Models, Heston, Smile Adjustments, Model Calibration

Carol Alexander
Chair of Risk Management,
ICMA Centre,
Henley Business School at the University of Reading,
Reading, RG6 6BA, UK.
Email: c.alexander@icmacentre.rdg.ac.uk

Andreas Kaeck
ICMA Centre,
Henley Business School at the University of Reading,
Reading, RG6 6BA, UK.
Email: a.kaeck@icmacentre.rdg.ac.uk

1 Introduction

Research on hedging vanilla options has important practical applications. In particular, market makers set prices so that their net profit from the trade, after deducting hedging costs, has positive expectation. More accurate hedging allows these traders to reduce their bid-ask spread, and thus increase their volume of trade. There is no constraint that the model used for hedging liquid options should be the same as the model used for pricing illiquid options. Indeed, there is a large literature on option-pricing models that focuses exclusively on their relative hedging performance, in particular, on their accuracy when hedging standard European (vanilla) equity index options.

This paper implements a variety of calibration methods for the stochastic volatility model of Heston (1993) with the aim of assessing their effect on the performance of standard and minimum-variance (MV) delta, delta-gamma and delta-vega hedging of vanilla options. Simple adjustments to the Black-Scholes-Merton (BSM hereafter, Black and Scholes (1973) and Merton (1973)) hedge ratios are used as a benchmark.

We use the Heston model because the research on its performance for hedging equity index options is already considerable, and robust results have been obtained using a variety of indices and sample periods. Kim and Kim (2004) show that Heston's model outperforms other stochastic volatility models in-sample, out-of-sample and for hedging, and (Kim and Kim (2005)) confirm this when jumps are added to the price process. Bakshi, Cao, and Chen (1997) use the Heston model to conclude that "*once stochastic volatility is modeled, adding the SI (stochastic interest rate) or the random jump feature does not enhance hedging performance any further.*" Alexander, Kaeck, and Nogueira (2009) find that the Heston model performs particularly well in a MV delta-gamma hedging exercise. The Heston model has also become a frequently used model in simulation studies that investigate hedging performance (Psychoyios and Skiadopoulos (2006), Poulsen, Schenk-Hoppé, and Ewald (2009), Branger, Schlag, Schneider, and Seeger (2008)).

Although the pleasing hedging performance of the Heston model, coupled with its ease of calibration, has established it as a standard stochastic volatility model for the investigation of hedging performance, there are two important issues. Firstly, several studies report that the Heston dynamics are grossly misspecified (e.g. Eraker, Johannes, and Polson (2003), Jones (2003), Broadie, Chernov, and Johannes (2007)). Shortcomings of the model are related to the existence of possible jumps in price and volatility. In fact, some authors advocate to leave the class of affine pricing models altogether.[1] An important consequence is that model misspecification can translate into inconsistencies between parameter estimates that are solely based on time series of underlying prices and parameter estimates that are inferred from option quotes.[2] Parameter estimates based on the underlying price process are often regarded as inferior, because they are based on historical information rather than on trader's beliefs about the evolution of the underlying price over the life of the option.[3] Altogether, the fact that different calibration techniques

[1]For example Christoffersen, Jacobs, and Mimouni (2010) report that non-affine models are superior for both in- and out-of-sample statistical tests.

[2]See for example Bakshi, Cao, and Chen (1997). Also, Broadie, Chernov, and Johannes (2007) give an illustrative example of how unrealistic the parameters of a misspecified model can be, when calibrated to a cross section of option prices (their Figure 3, p. 1464).

[3]One could also infer parameter values from options prices and underlying prices simultaneously, as in Chernov and Ghysels (2000), Pan (2002), Eraker (2004) and others. However such approaches are often

can lead to very different parameter values for the Heston model, leads one to the important question of whether the data source for the model calibration matters.

Secondly, most of the hedging literature relies on daily calibration of the Heston model to a cross section of option prices. Although common, this approach is inconsistent, because within the model all parameters are assumed to be time-homogeneous. Thus the model that has usually been tested in the empirical hedging literature is not the Heston model itself, but an extension where calibrated parameters are allowed to vary over time. A correct implementation of the model should separate out an estimation period and use one (constant) set of parameters throughout the out-of-sample hedging period.[4] Alternatively, if the calibrated parameters are systematically related to the underlying price process, adjustments of hedge ratios are necessary (see Alexander, Kaeck, and Nogueira (2009)).[5]

Our work represents a new departure in empirical hedging studies in the literature. We focus not on the hedging model, but on the model calibration technique itself. We provide evidence on the relative importance, for the purpose of option hedging, of the data source (options or underlying prices) and the quality of fit. Considering various calibration approaches, with daily and pre-sample calibration to option prices, we compare the hedging performance with that obtained when parameters are inferred purely from the underlying index. Our empirical results are very comprehensive: they cover a large sample (from 2 January 2002 to 31 December 2008) of delta, delta-gamma and delta-vega hedging, using both standard and MV approaches. On the methodological side, we introduce a procedure to calibrate option pricing models simultaneously to option and implied volatility index data, which is very efficient because it considerably reduces the dimensionality of the optimization problem.

We proceed as follows: Section 2 presents an in-depth discussion of the Heston model and the simple adjustments to the BSM model that are used as benchmarks; Section 3 describes the three different calibration techniques applied to the Heston model; Section 4 defines the hedging strategies; Section 5 presents the empirical results; and Section 6 concludes.

2 Models

2.1 Heston Model

The Heston model assumes that on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t\in\mathbb{R}})$, where $\mathbb{Q}$ denotes the objective (real world) probability measure, the equity index $S(t)$ and its variance $V(t)$ solve

$$dS(t) = \mu S(t)\,dt + \sqrt{V(t)}S(t)\,dW^s(t) \tag{1}$$

$$dV(t) = [\theta - \kappa V(t)]\,dt + \xi\sqrt{V(t)}\,dW^v(t) \tag{2}$$

extremely time consuming and are difficult to handle for a very large data set such as ours.

[4] Of course, such an approach compromises the fit of the pricing model and models that are calibrated in this way cannot be used for option pricing.

[5] We do not include these hedge ratios on our study because each calibration requires a large number of integrals to be evaluated using Fourier transform methods, which is computationally infeasible on a large sample.

where $W^s(t)$ and $W^v(t)$ are two correlated Brownian motions with $\mathbb{E}\left[W^s(t)\, W^v(t)\right] = \varrho t$, μ is the constant drift of the index, κ the mean reversion speed of the variance, θ/κ is the long term variance and ξ is commonly referred to as the *volatility-of-volatility (vol-of-vol)*.[6] We collect all structural parameters in the vector $\mathbf{\Phi} = \{\mu, \kappa, \theta, \xi, \varrho\}$.

This model is in line with many stylized facts reported for equity indices. Firstly it accommodates increasing excess kurtosis in returns via increasing ξ and $|\varrho|$. This feature is important, because it produces implied volatility graphs that have the well-known smile pattern. Also, a negative correlation between the underlying and its variance creates negative skewness in returns, an effect that is often termed the *leverage effect* (Black (1976)). It implies that falling prices tend to be accompanied by increasing variance and it leads to skewed or 'smirk' shaped implied volatilities with respect to strike. In addition the model allows variance to tend to a long-term value. Thus variance can fluctuate over time, but it will never wander away too much from its equilibrium value to which it is pulled back in the long run. This results in more realistic term structure behavior of volatility compared to simpler drift specifications. The main drawback, however, is that sample paths generated by the Heston model are continuous and no sudden jumps in the price process can be created. Although econometric research on the data generating process of equity indices favors models with jumps, the empirical hedging literature finds little evidence that including jumps improves the hedging performance.

One of the main reasons why the Heston model has become popular is its ability to express prices of standard vanilla options in nearly closed form. This requires the transition from the objective probability measure $\mathbb{Q}$ to the riskneutral measure $\widetilde{\mathbb{Q}}$. For a linear variance risk premium the structure of the model remains identical after a change of measure:

$$\begin{aligned} dS(t) &= rS(t)\,dt + \sqrt{V(t)}S(t)\,d\widetilde{W}^s(t) \\ dV(t) &= \left[\theta - \widetilde{\kappa}V(t)\right]\,dt + \xi\sqrt{V(t)}\,d\widetilde{W}^v(t), \end{aligned}$$

where $\widetilde{W}^s(t)$ and $\widetilde{W}^v(t)$ are two Brownian motions under $\widetilde{\mathbb{Q}}$, still having correlation ϱ, and r denotes the risk-free interest rate. Structural parameters are now $\mathbf{\Phi}^{rn} = \{\widetilde{\kappa}, \theta, \xi, \varrho\}$ where the variance risk premium is $\lambda = \kappa - \widetilde{\kappa}$. Absence of arbitrage requires θ, ξ and ϱ to be the same under both measures, thus no matter whether parameters are inferred from the evolution of the equity index (and thus under the objective measure $\mathbb{Q}$) or from its option prices (thus under the riskneutral measure $\widetilde{\mathbb{Q}}$), parameters should be consistent.

Vanilla option prices can be calculated by inverting the characteristic function as shown in Heston (1993), Carr and Madan (1999), Lewis (2000) or Duffie, Pan, and Singleton (2000). Adopting the methodology from Carr and Madan (1999), this leads to a time-t vanilla call price (with maturity T, residual time to maturity $\tau = T - t$, and strike K) given by the following integral, which can be evaluated by standard numerical integration methods or the fast Fourier transform (FFT):

$$C(t,T,K) = \frac{\exp\left(-\alpha \log K\right)}{\pi} \int_0^\infty e^{-iv\log K} \frac{e^{-r\tau} f(v - (\alpha+1)i, t, T)}{\alpha^2 + \alpha - v^2 + i(2\alpha+1)v}\, dv,$$

[6] The evolution of the variance can also be written as $dV(t) = \kappa\left[m - V(t)\right]\,dt + \xi\sqrt{V(t)}\,dW^v(t)$, thus explicitly providing a parameter (here $\sqrt{m}$) that accounts for the long-run value of volatility. We use the alternative specification because it simplifies notation when including risk premia.

where α denotes a dampening factor (such that the $(\alpha+1)$st moment of $\log S(T)$ exists) and $f(u,t,T) = \mathbb{E}\left[e^{iu\log S(T)}\middle|\mathcal{F}_t\right]$ is the characteristic function of the log stock price. Under the objective probability measure $\mathbb{Q}$ this is given by:

$$
\begin{aligned}
f(u,t,T) \quad = \quad & \exp\left\{iu\left(\log S(t) + \mu(T-t)\right)\right\} \\
& \times \exp\left\{\frac{\theta}{\xi^2}\left[(\kappa - \varrho\xi iu - d)(T-t) - 2\log\left(\frac{1-ge^{-d(T-t)}}{1-g}\right)\right]\right\} \\
& \times \exp\left\{\frac{(\kappa-\varrho\xi iu - d)(1-e^{-d(T-t)})}{\xi^2(1-ge^{-d(T-t)})}V(t)\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
d &= \sqrt{(\varrho\xi ui - \kappa)^2 + \xi^2(iu + u^2)}, \\
g &= \frac{\kappa - \varrho\xi iu - d}{\kappa - \varrho\xi iu + d},
\end{aligned}
$$

and $i = \sqrt{-1}$. Under the risk-neutral measure $\widetilde{\mathbb{Q}}$, we just replace μ by the risk free rate r and κ by its risk-neutral counterpart $\widetilde{\kappa}$.

2.2 Smile Adjustments and Sticky Models

We use three adjustments to the Black-Scholes-Merton (BSM) model as benchmarks against which to compare the performance of the Heston model. This helps to understand the significance of the calibration effect on the Heston model's hedging performance. Moreover, we contribute to the discussion on whether stochastic volatility models can outperform simple hedging models which do not specify explicit dynamics for the underlying price. This question has been addressed by several authors (for example Dumas, Fleming, and Whaley (1998), Vähämaa (2004) or Kim (2009)). So far, empirical studies have been limited to relatively short samples, and results have been inconclusive.

Smile adjusted hedge ratios are based on the assumption that the underlying price process is a geometric Brownian motion, but the constant volatility of that process depends on the option that is being priced. Since there is only one underlying price for all options these so-called 'sticky models' use deliberately inconsistent assumptions about the price process. Nevertheless, they offer a pragmatic approach to hedging and have become very popular with option traders.

Two such approaches have been termed 'sticky-strike' and 'sticky-moneyness'. In the sticky strike approach each option is allowed to have a different implied volatility.[7] Under the sticky-moneyness assumption, the implied volatility of an option is assumed constant with respect to its moneyness, rather than its strike. Thus a spot price move leads to a change in the implied volatility of the option.

To implement the sticky-moneyness hedging model we parameterize the implied volatility to be a cubic function of moneyness $m = \log(K/F)/\sqrt{\tau}$, where F is the current index

[7] This approach is sometimes also called the 'practitioner Black-Scholes model' because it results in calculating hedge ratios by using the BSM formula with its implied volatility surface. This is in stark contrast to the original BSM model where hedge ratios for all options are calculated with a unique volatility parameter.

futures price (with the same maturity T as the option).[8] That is,

$$IV(m, \boldsymbol{\alpha}_t) = \alpha_{t,0} + \alpha_{t,1}\, m + \alpha_{t,2}\, m^2 + \alpha_{t,3}\, m^3 \tag{3}$$

where the parameters $\boldsymbol{\alpha}$ are calibrated by minimizing the objective function (4) defined below. Then the hedge ratios for the sticky moneyness assumption can be calculated by a standard central difference approach. For instance for the option delta (with respect to the futures price) we shift the futures price and the implied volatility consistently:

$$\frac{BSM\left(F_{up}, K, t, T, IV(m_{up}, \boldsymbol{\alpha}_t)\right) - BSM\left(F_{down}, K, t, T, IV(m_{down}, \boldsymbol{\alpha}_t)\right)}{F_{up} - F_{down}},$$

where BSM denotes the BSM price (Black (1976)) and

$$m_{up/down} = \log\left(K/F_{up/down}\right)/\sqrt{\tau}.$$

The same central difference methodology can be applied to obtain a gamma hedge ratio. A similar approach, but based on the local volatility surface, has been adopted by Engelmann, Fengler, and Schwendner (2006).

A third, but more complex approach, is labeled 'sticky tree' or the 'local volatility' model. This approach goes back to Derman and Kani (1994), Dupire (1994) and Rubinstein (1994). There is a wide literature on the implementation of such models which often includes the numerical solution of the Dupire partial differential equation. However Derman, Kani, and Zou (1996) and Coleman, Kim, Li, and Verma (2001) provide theoretical justifications to approximate the local volatility delta hedge ratio by adjusting the practitioner BSM delta by the slope of the volatility smile:

$$\delta_{BSM} + \nu_{BSM} \frac{\partial IV(m, \boldsymbol{\alpha})}{\partial m} \frac{\partial m}{\partial K},$$

where δ_{BSM} and ν_{BSM} denote the delta and vega, respectively, in the BSM model when using implied volatilities, i.e. the sticky-strike delta and vega.[9] Similarly the gamma follows as the first derivative of the above delta. For these hedge ratios, partial derivatives of the smile are taken from the interpolated volatility surface in Equation (3).

3 Model Calibration

3.1 Local vs Global Calibration

Here we describe the two main approaches to stochastic volatility model calibration that have been adopted in the empirical literature. Daily calibration to the cross section of

[8] Note that the index futures is the hedging instrument. We also tested more general implied volatility-moneyness specifications that allow for additional dependence on the time-to-maturity of the option and/or mixed terms including both moneyness and time-to-maturity, but found that a simple cubic polynomial works as well as more complex specifications.

[9] We follow Engelmann, Fengler, and Schwendner (2006) by defining the vega in all sticky models as the BSM vega (using implied volatilities), thus it represents a parallel shift in the volatility surface.

option prices, where parameters and the latent state variable are simultaneously inferred from quotes on one trading day, consists of optimizing an objective function such as

$$\left(\widehat{\mathbf{\Phi}^{rn}}_t, \widehat{V}(t)\right) = \arg\min \frac{1}{N_t} \sum_{i=1}^{N_t} \left(P^{mo}(t, K_i, T_i, \mathbf{\Phi}_t^{rn}, V(t)) - P^{ma}(t, K_i, T_i)\right)^2, \tag{4}$$

where P^{mo} and P^{ma} denote the model and market prices of the options in the calibration set, T_i is the maturity of the ith option, N_t refers to the number of options used for the calibration at time t, and trading days in the calibration sample are denoted $t = 1, ..., K$. Repeated application of this calibration technique to panel data on option prices yields a time-series for each of the structural parameters $\widehat{\mathbf{\Phi}_t^{rn}}$, and for the latent state variable $\widehat{V}(t)$.

A detailed discussion on the choice of calibration objective can be found in Christoffersen and Jacobs (2004). Instead of using an absolute price metric one could employ a relative price metric, or minimize the squared implied volatility errors. Weights that emphasize the errors on the most liquid options could also be applied. We opt for (4) to facilitate the use of standard deviation as a meaningful and consistent out-of-sample objective for our hedging experiment. Also, this approach has been adopted for example in Bakshi, Cao, and Chen (1997), Kim and Kim (2005) and Alexander and Nogueira (2007) and it is well-known that it produces a reasonable fit to option prices on any given day. In the following we refer to this calibration approach as *local calibration.*

However, the local calibration approach violates one of the basic assumptions of the model: that the structural parameters are time-homogeneous. If calibrations are stable over time this might not be a major concern, as minor variations can be attributed to price discreteness and other microstructure effects. However, this is rarely the case for equity indices. We therefore consider an alternative, which circumvents the problem of time-varying parameters, where we reserve a pre-sample period for estimating the latent state variable and one set of structural parameters.

We refer to this approach as *global calibration.* Here, the equivalent (un-weighted price metric) optimization problem to (4) becomes:

$$\left(\widehat{\mathbf{\Phi}^{rn}}, \left\{\widehat{V}(t)\right\}_{t=1}^{K}\right) = \arg\min \frac{1}{\sum N_t} \sum_{t=1}^{K} \sum_{i=1}^{N_t} \left(P^{mo}(t, K_i, T_i, \mathbf{\Phi}^{rn}, V(t)) - P^{ma}(t, K_i, T_i)\right)^2.$$

Several optimization techniques for global calibration have been adopted, among these the simulated method of moments (adopted for example in Bakshi, Cao, and Chen (2000)) or the bootstrap procedure in Broadie, Chernov, and Johannes (2007). One of the main challenges to address with this optimization is the dimensionality of the problem, as we need to infer not only the structural parameters but also the spot variance over the whole estimation period. In addition, if the model is to be applied out-of-sample, a filtering algorithm needs to be implemented to extract the unobservable variance on the out-of-sample days.

We now introduce a new approach to global calibration that solves this dimensionality problem by utilizing some well-known results from the model-free volatility literature that link the (unobservable) state variable $V(t)$ with the (observable) underlying volatility index (VI) having some constant time to maturity τ_v. Applying the results of Britten-Jones and

Neuberger (2000) to the Heston model, one obtains:

$$\mathrm{VI}^2(t,\tau_v) = \frac{\theta}{\widetilde{\kappa}}\left(1 - \frac{1-e^{-\widetilde{\kappa}\tau_v}}{\widetilde{\kappa}\tau_v}\right) + \frac{1-e^{-\widetilde{\kappa}\tau_v}}{\widetilde{\kappa}\tau_v}V(t). \tag{5}$$

This closed-form relation between volatility indices and the spot variance can be used in the calibration, and circumvents the need to estimate the spot variance directly. In addition, there is no need to implement a filtering algorithm for the out-of-sample period, because spot volatilities become directly observable once the structural parameters are known.

Inverting (5) yields the following function for spot variance, in terms of the volatility index:

$$V(t) \quad = \quad \frac{e^{\widetilde{\kappa}\tau_v}\,\mathrm{VI}^2(t,\tau_v)\,\widetilde{\kappa}^2\tau_v + \theta\left(e^{\widetilde{\kappa}\tau_v}(1-\widetilde{\kappa}\tau_v)-1\right)}{\left(e^{\widetilde{\kappa}\tau_v}-1\right)\widetilde{\kappa}}. \tag{6}$$

The dimensionality of the optimization can now be reduced by using the volatility index directly in the optimization routine, thus using Eqn (6) we obtain a much simpler optimization problem as follows:

$$\widehat{\mathbf{\Phi}^{rn}} = \arg\min \frac{1}{\sum N_t}\sum_{t=1}^{K}\sum_{i=1}^{N_t}\left(P^{mo}(t,K_i,T_i,\mathbf{\Phi}^{rn},V(t,\mathbf{\Phi}^{rn},\mathrm{VI}(t))) - P^{ma}(t,K_i,T_i)\right)^2. \tag{7}$$

3.2 Time-Series Consistency

A concern that is common to both the local and the global calibration approach is that they ignore the information from the time-series of the underlying. It is often argued that calibrating to option prices has the advantage of being a forward-looking strategy. However when a pure calibration approach is adopted the implied parameters are usually far away from their time-series counterparts (Bakshi, Cao, and Chen (1997)). This is problematic since for $\mathbb{Q}$ and $\widetilde{\mathbb{Q}}$ to be absolutely continuous only the mean reversion speed is allowed to change across measures.

To reduce the effect of model misspecification on risk premia estimates, this observation motivated Broadie, Chernov, and Johannes (2007) to implement a two stage calibration methodology. First they fix parameters that are restricted across measures to the time-series estimates of Eraker, Johannes, and Polson (2003) and then they calibrate the remaining parameters and the spot volatility to option prices.[10] Since a change of measure for pure jump processes is very flexible and requires no restriction on parameters, in a jump-diffusion framework there are still enough degrees of freedom to obtain a good fit to option prices even after fixing some parameters to their time-series values. However, in the Heston model we are left with only one free parameter. For this reason, we adopt a more pragmatic procedure: First we estimate the parameters in $\mathbf{\Phi}$ from the time-series of FTSE 100 returns, and then we translate these estimates into riskneutral parameters by making assumptions on the variance risk premium λ. This approach has also been adopted by Dotsis, Psychoyios, and Skiadopoulos (2007) for the VIX index.

[10] It should be noted that fixing some parameters to reasonable values before calibration is quite common (see Bates (2000), Broadie, Chernov, and Johannes (2007) and others.)

Estimating the parameters of the Heston model from return observations of the FTSE index is not straightforward, because of the latent nature of volatility. This latent variable makes the application of the maximum likelihood principle for estimation somewhat burdensome. Instead, Bayesian methods have become popular, mainly because of their advantages when it comes to estimating unobserved state variables such as jumps, jump sizes or volatility. Jacquier, Polson, and Rossi (1994) pioneer Monte Carlo Markov Chain (MCMC) algorithms to estimate discrete time stochastic volatility models and Eraker, Johannes, and Polson (2003) adopt a similar procedure to continuous time finance models (including the Heston model). The authors show that MCMC applied to a time-discretized version of Eqn (1) and (2) yields accurate inference about the parameters.

MCMC methods require the derivation of complete conditional distributions. Using a time-discretized version of the Heston process it is straightforward to derive these for μ, κ and θ. For the correlation ϱ and the volatility-of-volatility parameter ξ, two different algorithms are used in the literature. Jacquier, Polson, and Rossi (2004) reparameterize the variance process which leads to known distributions for both parameters, whereas the methodology in Eraker, Johannes, and Polson (2003) requires a Metropolis step to update the correlation. We have experimented with both algorithms but for brevity only reporting results based on the method of Jacquier, Polson, and Rossi (2004), which circumvents the use of a Metropolis algorithm. Complete conditional distributions for the variance lead to densities of unknown form, which can be sampled by a hybrid Metropolis-within-Gibbs step. For more details on the algorithm for the Heston model and the Metropolis step (and MCMC in general) we refer the interested reader to Eraker, Johannes, and Polson (2003) or Johannes and Polson (2006). To start the procedure, it remains to fix the prior information. We use similar prior distributions and parameters as Eraker, Johannes, and Polson (2003). There is, in general, little information in the prior distributions and hence we put as much weight as possible to the data. More specific details on our implementation of this algorithm, including the prior information, are given in the appendix.

4 Hedging Strategies

As usual, we define the standard delta and gamma of an option to be the first and second partial derivatives of its model price with respect to the underlying asset price. In a complete market with no frictions, continuously rebalanced delta hedging should remove all risk from the option position. But in practice transaction costs and trading limitations render continuous rebalancing impossible and one has to resort to using discrete hedging intervals. We shall rebalance daily, and so the resulting residual risk should be reduced by the addition of a gamma or vega hedge, which requires an investment in a second option on the same underlying. When the option is priced according to an underlying price process with constant volatility, the option vega is the first partial derivative of the option price with respect to this volatility. But in models which render the market incomplete a standard delta hedge would leave residual risk even under continuous rebalancing. In the Heston model, where the residual risk arises because of stochastic volatility, a delta hedge has to be complemented by a vega hedge to remove risks completely. Under the Heston model, which has a stochastic volatility, we define the option vega to be the first derivative of the Heston model price with respect to the spot volatility.

Alternative tractable hedging strategies have been introduced that minimize the variance of the local hedging error. These hedge ratios have been labeled *minimum variance* or *locally risk minimizing hedge*; they have been studied for example in Bakshi, Cao, and Chen (1997) and Poulsen, Schenk-Hoppé, and Ewald (2009). In stochastic volatility models this hedge ratio takes into account all the risks arising from the movements in the Brownian motion $dW^s(t)$ and thus partially hedges risks due to a stochastic volatility that is correlated with $dW^s(t)$. Minimum-variance (MV) gamma hedging has been proposed for stochastic volatility models in order to partly account for gamma risk that arises from the volatility process. Therefore, in our empirical hedging study we use MV delta and delta-gamma hedges as well as standard delta, delta-gamma and delta-vega hedging strategies. For a derivation of MV delta and gamma for the Heston model, see Alexander and Nogueira (2007).

To fix notation assume that a hedge is rebalanced at discrete intervals of length Δt. In a delta hedging strategy, at every point in time t, the short position in a call or put option (with price here denoted simply by O) is complemented by an investment in $X_S(t)$ shares of the underlying, where $X_S(t)$ is equal to the delta hedge ratio of the pricing model. As a hedging instrument we employ the futures with the maturity T' closest to the maturity T of the option (and in most cases, $T' = T$). Thus we translate the sensitivity $X_S(t)$ into $X_{F_{T'}}(t)$, where $F_{T'}(t)$ denotes the price of the closest futures contract. This requires the calculation of hedge ratios with respect to the hedge instrument, which can be easily derived from chain rule of calculus. In addition to the futures, we add an investment of $X_r(t)$ in the risk-free rate, so that the hedge portfolio has a value of zero at initiation.

After one time step Δt we obtain a hedging error of

$$e^{-rT'} X_{F_{T'}}(t)\left[F_{T'}(t+\Delta t) - F_{T'}(t)\right] + e^{r\Delta t} X_r(t) - \left[O(t+\Delta t, T) - O(t,T)\right] \tag{8}$$

for every option and every rebalancing day in our sample. Similarly delta-gamma and delta-vega hedging performance can be assessed for every option price in our sample, by defining a hedging error that includes the position in a second option $X_{\bar{O}}(t)$:

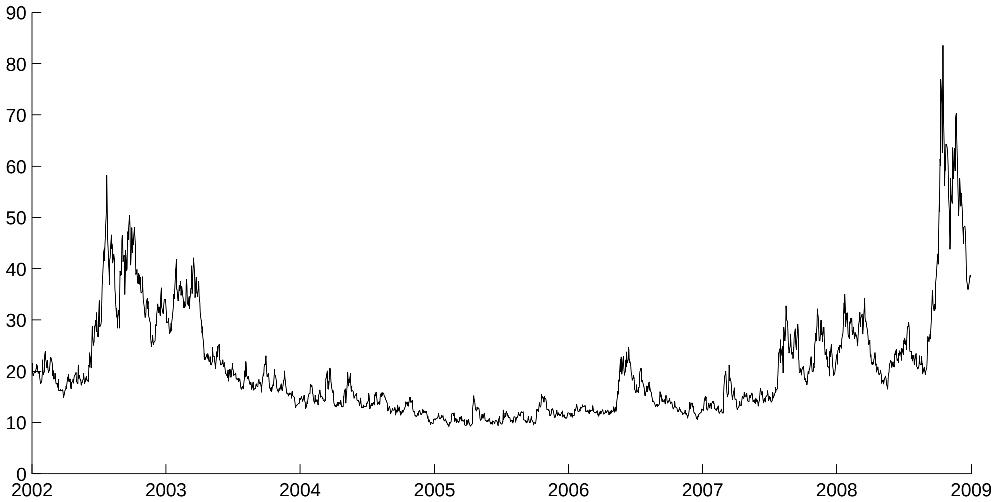
$$\begin{aligned} &e^{-rT'} X_{F_{T'}}(t)\left[F_{T'}(t+\Delta t) - F_{T'}(t)\right] + e^{r\Delta t} X_r(t) \\ &+X_{\bar{O}}(t)\left[\bar{O}(t+\Delta t, T) - \bar{O}(t,T)\right] - \left[O(t+\Delta t, T) - O(t,T)\right], \end{aligned} \tag{9}$$

where $X_{\bar{O}}(t)$ is determined by the ratio of the gamma (or vega) of the hedging option and the option to be hedged. Such a hedge can be implemented with any option that exhibits a non-zero gamma (or vega).

5 Empirical Results

In this section we first describe the data, then we present the calibration results for the Heston model, and finally we report the hedging results for the Heston model and the three smile adjustments of Section 2. Hedging results are for standard delta, delta-gamma and delta-vega hedging under all models, and for MV delta and delta-gamma hedging under the Heston model.

Figure 1: **VFTSE Index.** This figure plots the evolution of the VFTSE index for 30 days to maturity for the sample period. The index is constructed using the VIX methodology and is normalised to a yearly standard deviation.



5.1 Data

We use daily closing prices of FTSE 100 options obtained from Euronext-LIFFE (London International Financial Futures Exchange) from 2 January 2002 until 31 December 2008. These contracts are European style options on the FTSE 100 equity index (ESX). The options can be classified into two distinct categories depending on their expiration month. The first category are options in the quarterly cycle, where the expiration day is the third Friday in March, June, September or December. In addition to the quarterly cycle, serial options are made available for trading such that the quarterly cycle is complemented by short term options that expire in non-cycle months. Prior to 2004, the next four cycle-months and two short term serial options are available for trading. For instance in the beginning of March, expiry months are March, April, May, June, September and December.[11] From January 2004 on, also long-term options with time to maturity of up to two years were added to the data set and the number of serial options was raised to three.[12] As regards the strike range, option quotes between far out-of-the money (OTM) and far in-the-money (ITM) are available where the quoted strikes typically range between around 70% and 120% of the futures price.

We apply standard filters to our data. First, we remove quotes that violate standard no-arbitrage conditions such as in Bakshi, Cao, and Chen (1997). Second, we remove data outside the moneyness range -0.5 and 0.3 as these options are infrequently traded. For the same reason we discard options with more than one year to maturity. Third, we refrain from using option quotes with less than five days to maturity as their prices are often heavily influenced by market-microstructure effects. We also plot all options in

[11] The expiry date is always the third Friday in the month.

[12] The long term options extend the quarterly expiration months beyond one year and consequently the number of different maturities reaches up to twelve.

implied volatility-moneyness space to filter out obvious recording errors. After applying these filters we are left with more than 700,000 prices. We further divide the sample into an in-sample calibration period (January 2002 until June 2005) and an out-of sample period (July 2005 until December 2008). This leaves more than 356,000 observations for our out-of-sample hedging exercise.

To explore the sample graphically, and because we shall need this series when implementing our global calibration procedure, Figure 1 depicts the volatility index derived from these option prices from 2002 until the end of 2008.[13] It is apparent that the in-sample and out-of-sample periods are generally quite similar in nature, including both tranquil and more volatile market regimes. Volatile periods are observed between mid 2002 and mid 2003, and especially during the last quarter of 2008 when, with the advent of the banking crisis, FTSE 100 volatility reached more than 80%. Before this, such high levels of implied volatilities had only been witnessed after the global stock market crash of 1987. This extreme event poses a challenge for any hedging model and provide a means to compare the performance of alternative hedges during both normal and crash market regimes.

Finally, we require a substitute for the risk-free interest rate. No standard proxy has emerged in the literature to date. While some authors prefer using maturity matched interest rates by interpolating the yield curve (for example Broadie, Chernov, and Johannes (2007), Alexander, Kaeck, and Nogueira (2009)), others employ a short-term interest rate as a substitute for the unobservable instantaneous short rate. For example, Bliss and Panigirtzoglou (2004) argue that interpolated interest rates are unlikely to represent realistic borrowing and lending rates for traders in this market, and thus use the three month LIBOR rate (11 am fixing as reported by Bloomberg) as an approximate risk free rate. In addition, short term rates are heavily affected by central bank interventions and are less liquid compared with the three month rate. As most of the options in the sample are short-term options, interest rates have little effect on prices. Nevertheless, we experimented with both maturity-matched rates and a constant interest rate. Since short term interest rates can become highly erratic under central bank interventions, we prefer to follow Bliss and Panigirtzoglou (2004) in using the three month LIBOR rate as a substitute for the risk free rate.

5.2 Calibration Results

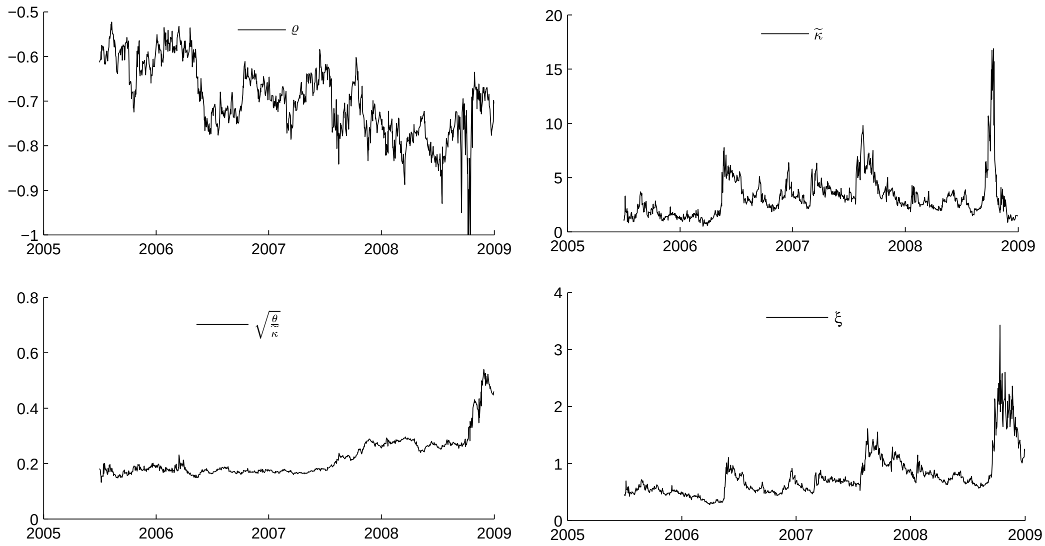
The Heston model is calibrated using local, global and time-series methods. For the global calibration we employ our new procedure as described in Section 3. For the time-series calibration we use the in-sample calibration period for calibration from 2 January 2002 to 30 June 2005. After estimating the spot index dynamics we apply a variance risk premium of $\lambda = 2$.[14]

The parameter estimates obtained from local calibration of the Heston model to our FTSE 100 option data set are depicted in Figure 2. The calibrated parameters change considerably from day to day, especially during the banking crisis. The day after the

[13] This index was constructed according to the VIX methodology. We are extremely grateful to Stamatis Leontsinis for constructing this series.

[14] Results for other values of the variance risk premium and for other in-sample calibration periods have been calculated but are not reported for brevity, since they do not change the qualitative nature of our results. They are available from the authors on request.

Figure 2: **Local Optimization.** This figure plots the evolution of the spot volatility (square root of the variance) and the structural parameters for daily calibrations of the Heston model to vanilla option prices on the FTSE 100 index, from June 2005 to December 2008. On each day in the sample the parameters are obtained by minimizing the objective function (4).



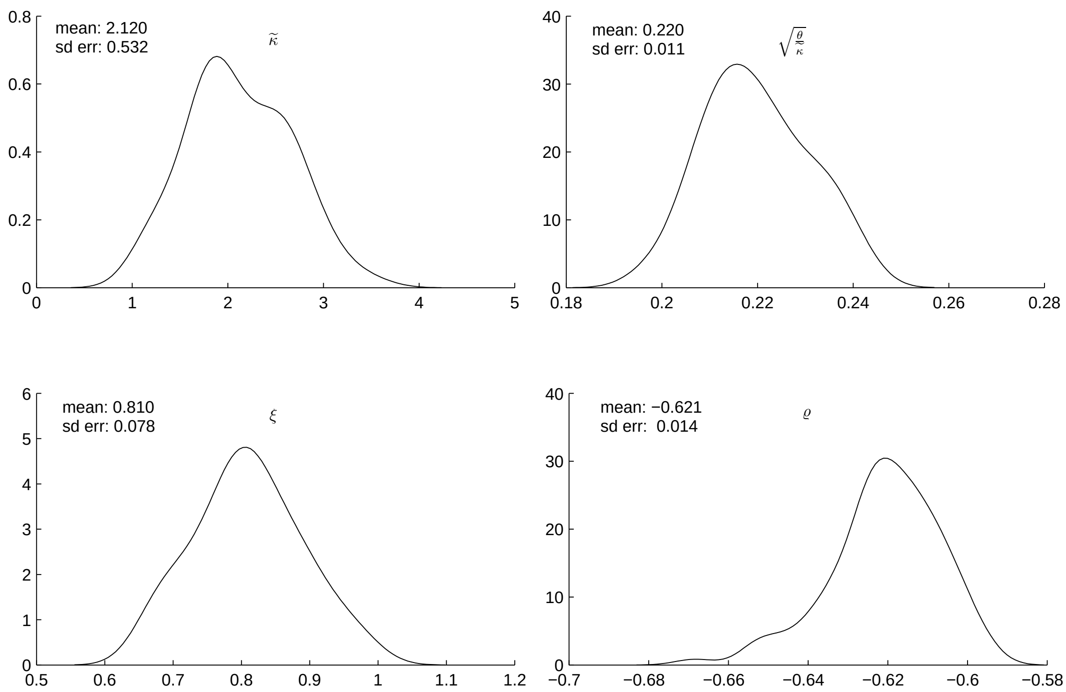
Lehman Brothers default the correlation coefficient hit its boundary value of minus one, thus creating a spike with exceptionally large and negative skewness in the risk-neutral density. There also seems to be a positive correlation between the mean reversion and vol-of-vol. The estimation of the long-run volatility is the most stable. It remained around 20% for most of the first half of the sample, but during the banking crisis its value adjusted to around 50%.

For our global calibrations of the Heston model, based on (7), we use VFTSE index shown in Figure 1 to derive the spot volatilities using (6). Then we follow the procedure employed by Broadie, Chernov, and Johannes (2007) and choose representative option data from 50 random Wednesdays during the calibration sample to estimate the structural parameters in $\widehat{\mathbf{\Phi}^{rn}}$. This procedure is repeated for 100 samples, which has the advantage of providing bootstrapped standard errors for the parameters, thus revealing how accurate the parameter estimates can be. The results are depicted by the kernel densities shown in Figure 3. In our hedging study we use the mean value, across all 100 samples, as our point estimate for each parameter. Note that, for each parameter, this mean value is similar to the average of its locally calibrated values over the calibration sample period. Hence, we can be sure that potential differences between the hedging performance of alternative calibration techniques is not due to substantially different parameter estimates.

Finally, for the time series calibration we use the daily returns on the FTSE spot index. For consistency, we have used the same sample as for the global calibration.[15]

[15]We also calibrated parameters using an extended sample covering 15 years (i.e. starting in January 1990) to satisfy ourselves that the calibration sample was large enough to produce robust parameter

Figure 3: **Global Optimization.** This figure plots Kernel densities of the parameter estimates obtained from 100 random samples of 50 Wednesdays in our calibration period. Parameters are obtained by optimizing Equation (7). Point estimates are obtained by using the mean over the 100 samples and standard errors are calculated from the standard deviation of the parameter estimates.



Applying the MCMC procedure, we use the posterior mean as our point estimate. This is in line with a quadratic loss function and is standard in the related literature. A possible alternative is to use the mode of the distribution but, as apparent from the results reported in Table 1, they are extremely close for all parameters. The results imply an estimate for the mean reversion parameter κ of 3.61, a long term volatility value of 17.82%, a vol-of-vol parameter ξ of 0.39 and an index-variance correlation parameter of -71%. Of course, these are slightly different from the results obtained by Eraker, Johannes, and Polson (2003) for the S&P index, due to the different behavior of the FTSE and the different sample period (Eraker, Johannes, and Polson (2003) include the market crash of 1987).

Our time-series calibration results exhibit similar stylized facts to those previously found when comparing estimates of S&P 500 dynamics that can be obtained from spot and option calibrations. That is, the vol-of-vol parameter differs significantly from its value obtained from a pure calibration to option prices. This observation was first made by Bakshi, Cao, and Chen (1997) and is often attributed to the misspecification apparent

estimates. For example, if volatility were relatively slow to mean revert, we might find that the parameters related to mean reversion of variance were quite different when calibrated to a much longer time series. However, the longer calibration sample had little effect on parameter values, and even less effect on our hedging results. Hence, these results are omitted for brevity, but are available from the authors on request.

Table 1: **Heston Model Calibration from Spot Data.** This table provides estimates of the structural parameters for the Heston model calibrated to FTSE 100 Spot data in the period from January 2002 until June 2005. We discarded the first 20,000 runs of the chain as a burnin and summarize the posterior with 20,000 draws.

Parameter	μ	κ	θ	ξ	ρ	**LT Vol**
From 01 January 2002 to 30 June 2005						
posterior mean	0.0007	3.6079	0.1112	0.3919	-0.7098	17.8203
posterior mode	0.0010	3.5269	0.1096	0.3914	-0.7166	17.5875
posterior stdev	0.0606	1.2251	0.0343	0.0421	0.0700	2.5575
5% percentile	-0.0988	1.7545	0.0578	0.3241	-0.8114	14.0799
95% percentile	0.1012	5.7775	0.1705	0.4620	-0.5846	22.3825

in the Heston model. Our option price implied parameter is 0.81 for the global calibration and has a similar average value over the local calibration sample. This is more than twice times the estimate obtained from spot data. Thus, using different calibration techniques might lead to very different results for the out-of-sample hedging exercise.

5.3 Hedging Results

On each trading day t between 1 July 2005 and 30 December 2008 we suppose that one option of every available strike and maturity has been written. Then we gamma hedge all these options using a single put option of relatively short maturity (typically in the region of 45 days) that is closest to at-the-money (ATM). Similarly, for the vega hedge we use a single put option of relatively long maturity (around 120 days), also closest to ATM. This because near ATM options have relatively large gammas and vegas, and puts are usually more liquid than calls.

Consistent with much of the empirical hedging literature we employ two different rebalancing intervals, one day and one week.[16] After this time the hedge portfolio is closed and the hedging error (8) or (9) is recorded. Then another portfolio of options is written and the procedure is repeated until all the data are exhausted. Under daily (weekly) rebalancing a total of 361,913 (73,078) options are written. We assume transactions costs have a similar effect on all hedges as our data includes only closing prices, no bid-ask spreads.

Our reported performance measure is the standard deviation of the hedge error, as this risk metric is consistent with our calibration objectives and the aim of hedging is to minimize risk.[17] To examine whether hedge performance is affected by market regime, we compute this standard deviation over two sub-samples: from 1 July 2005 to 30 June 2008, and from 1 July 2008 to 31 December 2008. Finally, to assess whether the performance depends on the moneyness of the option, we report results for all options and also disaggregated results for sub-groups of options with moneyness buckets $[-0.5, -0.3)$, $[-0.3, -0.1)$,$[-0.1, 0.1)$ and $[0.1, 0.3]$, where moneyness is defined as before.

[16]We follow standard practice in the empirical option literature and use non-overlapping Wednesday data for the weekly exercise.

[17]We have also computed average hedging errors and the mean absolute hedging error but have not included these to reduce the size of the results tables. Again, they are available from the authors on request.

Table 2: **Hedging Results - One day rebalancing.** This table reports the hedge error standard deviation for daily rebalancing, followed by its bootstrapped standard error (in parentheses) and 1% and 99% percentiles. Moneyness is defined in terms of $m = \log(K/F)/\sqrt{\tau}$ and given in the first column. The models are abbreviated as TS (time-series calibration with $\lambda = 2$), Local (parameters from local calibration), Global (parameters from global calibration), ST (sticky tree), SM (sticky moneyness), SS (sticky strike) and UH (unhegded position).

	TS		**Local**		**Global**		**ST**	**SM**	**SS**	**UH**
	Partial	MV	Partial	MV	Partial	MV				
Delta Hedge										
All Options	10.27	7.90	10.67	7.09	10.12	7.71	6.72	10.68	7.36	41.76
std err	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.04)	(0.04)	(0.05)	(0.04)	
1%-perc.	10.15	7.80	10.55	7.01	10.01	7.62	6.64	10.55	7.26	
99%-perc.	10.38	8.00	10.81	7.20	10.25	7.81	6.81	10.81	7.47	
-0.5 to -0.3	5.39	5.43	6.42	5.06	5.88	5.37	4.60	6.15	4.99	41.72
-0.3 to -0.1	7.84	7.34	10.16	6.93	9.44	7.13	6.61	10.11	7.22	39.20
-0.1 to 0.1	11.13	8.32	12.98	8.11	12.19	8.70	7.83	12.93	8.55	37.69
0.1 to 0.3	13.71	9.40	10.73	7.29	10.63	8.53	6.78	11.00	7.49	48.53
Delta-Gamma Hedge										
All Options	8.44	3.88	7.92	2.53	7.37	3.00	5.62	9.43	4.70	41.76
std err	(0.13)	(0.02)	(0.06)	(0.02)	(0.08)	(0.01)	(0.09)	(0.15)	(0.05)	
1%-perc.	8.13	3.84	7.78	2.49	7.2	2.97	5.43	9.08	4.59	
99%-perc.	8.73	3.91	8.07	2.57	7.56	3.04	5.83	9.79	4.81	
-0.5 to -0.3	7.01	3.15	4.62	2.76	4.97	2.93	5.89	5.44	3.92	41.72
-0.3 to -0.1	8.73	3.10	5.29	2.56	6.11	2.68	6.06	6.35	4.58	39.20
-0.1 to 0.1	8.41	2.84	7.42	2.05	7.83	2.34	4.99	8.09	4.81	37.69
0.1 to 0.3	9.16	5.76	11.84	2.81	9.29	3.94	5.66	14.63	5.22	48.53
Delta-Vega Hedge										
All Options	3.69		2.44		2.69		2.86	2.37	2.31	41.76
std err	(0.01)		(0.02)		(0.02)		(0.02)	(0.02)	(0.02)	
1%-perc.	3.66		2.40		2.65		2.80	2.33	2.26	
99%-perc.	3.73		2.48		2.73		2.91	2.40	2.35	
-0.5 to -0.3	2.97		2.52		2.65		3.23	2.60	2.46	41.72
-0.3 to -0.1	3.51		2.64		2.65		2.89	2.51	2.55	39.20
-0.1 to 0.1	2.69		2.11		2.33		2.15	2.04	2.02	37.69
0.1 to 0.3	5.16		2.53		3.13		3.23	2.39	2.25	48.53

Table 3: **Hedging Results - One week rebalancing.** This table reports the hedge error standard deviation for weekly rebalancing, followed by its bootstrapped standard error (in parentheses) and 1% and 99% percentiles. Moneyness is defined in terms of $m = \log(K/F)/\sqrt{\tau}$ and given in the first column. The models are abbreviated as TS (time-series calibration with $\lambda = 2$), Local (parameters from local calibration), Global (parameters from global calibration), ST (sticky tree), SM (sticky moneyness), SS (sticky strike) and UH (unhegded position).

	TS		**Local**		**Global**		**ST**	**SM**	**SS**	**UH**
	Partial	MV	Partial	MV	Partial	MV				
Delta Hedge										
All Options	22.95	16.97	23.62	14.73	22.74	15.88	13.95	23.51	17.02	84.29
std err	(0.15)	(0.14)	(0.18)	(0.12)	(0.17)	(0.12)	(0.12)	(0.17)	(0.14)	
1%-perc.	22.61	16.66	23.23	14.45	22.38	15.57	13.69	23.13	16.70	
99%-perc.	23.28	17.3	24.04	15.01	23.15	16.17	14.21	23.93	17.33	
-0.5 to -0.3	12.53	12.17	14.27	11.40	13.48	11.98	10.70	13.69	11.75	84.35
-0.3 to -0.1	18.75	16.58	23.09	15.73	22.22	16.09	14.88	23.01	17.36	78.42
-0.1 to 0.1	26.17	18.59	29.68	17.38	28.56	18.36	16.31	29.43	20.45	76.34
0.1 to 0.3	28.34	18.44	21.52	12.22	20.89	15.03	11.92	21.84	15.40	98.22
Delta-Gamma Hedge										
All Options	15.23	7.69	16.08	5.00	14.30	5.77	9.98	16.90	9.42	84.29
std err	(0.22)	(0.05)	(0.16)	(0.07)	(0.15)	(0.05)	(0.24)	(0.29)	(0.11)	
1%-perc.	14.74	7.58	15.72	4.85	13.95	5.66	9.41	16.23	9.18	
99%-perc.	15.75	7.82	16.46	5.15	14.62	5.89	10.51	17.61	9.68	
-0.5 to -0.3	12.87	6.57	9.51	6.01	9.11	6.18	9.80	10.08	8.35	84.35
-0.3 to -0.1	15.32	6.68	10.48	5.61	10.89	5.74	10.85	11.26	9.06	78.42
-0.1 to 0.1	15.39	5.51	14.94	3.60	15.63	4.23	9.31	14.97	9.56	76.34
0.1 to 0.3	16.58	11.07	24.24	4.91	18.40	6.99	9.96	25.90	10.35	98.22
Delta-Vega Hedge										
All Options	7.41		5.17		5.43		5.57	4.70	4.69	84.29
std err	(0.06)		(0.07)		(0.06)		(0.07)	(0.05)	(0.05)	
1%-perc.	7.29		5.01		5.30		5.40	4.59	4.57	
99%-perc.	7.57		5.34		5.58		5.74	4.82	4.83	
-0.5 to -0.3	6.23		5.69		5.75		6.39	5.56	5.52	84.35
-0.3 to -0.1	7.51		6.04		5.85		6.28	5.27	5.40	78.42
-0.1 to 0.1	5.26		3.97		4.42		3.99	3.63	3.59	76.34
0.1 to 0.3	9.98		5.08		5.82		5.74	4.50	4.36	98.22

A Daily rebalancing

The performance of the different delta, delta-gamma and delta-vega hedges with daily rebalancing is reported in detail in Table 2. We concentrate first on the delta hedging exercise. The results can be summarized as follows:

– Not surprisingly, given the theoretical results of Bates (2005) and Alexander and Nogueira (2007), the sticky-moneyness (SM) and standard Heston hedges (local calibration) yield very similar results. These two hedges provide the worst performance overall.
– The standard Heston model delta hedges perform worse than MV hedges, irrespective of the calibration method. This finding confirms the results of Alexander and Nogueira (2007) and Poulsen, Schenk-Hoppé, and Ewald (2009) and is due to a residual risk arising from uncertainty in volatility. Therefore, we concentrate only on MV hedge ratios in the following.
– In general, the best Heston model delta hedging results are obtained from MV hedging with locally calibrated parameters. Time-series parameter estimates yield the highest standard error so the inclusion of option prices in the calibration procedure is important. This result is robust with respect to the moneyness of the options, the only exception being the ATM category, where the time-series calibration outperforms the global calibration.
– The sticky-tree (ST) delta hedge ratios out-perform the Heston deltas for all moneyness buckets and all calibration techniques, whereas the the practitioner BSM model (i.e. the sticky-strike (SS) adjustment) deltas outperform the locally-calibrated MV deltas only for low strike options (i.e. those in the $[-0.5, 0.3)$ moneyness range.)
– As an interesting consequence of our results, the SS adjustment ranks between the global and the local calibration implementation. Hence the ranking changes depending on the calibration technique chosen.

To assess the statistical significance of the differences in the reported hedging error standard deviations, we bootstrapped confidence intervals for these in the all-options category. The standard error and the 1% and 99% percentiles of the bootstrapped distribution are displayed below the hedging error standard deviation Table 2. The results confirm that the differences between all models are significant. In particular, the standard deviations of the hedging errors based on the locally calibrated parameters are significantly greater than those of the ST adjustment, but significantly smaller than those of the practitioner BSM model (SS), because their 1-99%-confidence intervals do not even overlap.

The results for the daily-rebalanced delta-gamma and delta-vega exercise are reported in the middle and lower section of Table 2. Our findings are summarized as follows:

– The addition of a gamma hedge reduces risk considerably under the Heston model, with standard deviations of about half the size of the delta-only hedged positions. The local calibration (with MV hedge ratios) is still by far the best performing strategy in the Heston model. This holds for all options and in all moneyness subcategories.
– MV hedging now significantly out-performs all the smile-adjusted models (except when when hedging very high strike options, using time-series calibration and only if we use partial derivative hedge ratios).

– The addition of a gamma hedge achieves much less risk reduction under the smile-adjusted models. In fact, it can even increase the hedging error variance in the ST model for low strike options. Interestingly, the ST and SM hedges now perform significantly worse than the practitioner BSM (SS) hedge.
– The worst delta-vega hedging results are obtained using time-series calibration of the Heston model. The other Heston model calibrations produce results very similar to the smile-adjusted delta-vega hedges. For the Heston delta-vega hedges, local calibration is still significantly better than global calibration, and ST now performs slightly worse than SS or SM.
– The addition of a vega hedge reduces risk considerably under the smile-adjusted models. Even the SM hedge now performs similarly to the locally calibrated Heston hedges.

B Weekly Rebalancing

Results for the weekly rebalancing strategies are given in Table 3. Overall, the qualitative results from the daily hedging exercise carry over to the weekly rebalancing frequency. In particular, in the delta hedging exercise the ST hedging errors are the least variable. The second best performance is from the locally calibrated Heston model, followed by the global and the time series calibrations. The SS hedge, and even more so the SM hedge, perform worse than any Heston model implementation. Bootstrapping results again confirms that the difference between the models is highly significant.

For the delta-gamma strategy, smile adjustments underperform the Heston model, just as they did with daily rebalancing. Again the locally calibrated Heston model has the best performance, with a hedging error standard deviation that is about one third of its value based on a pure delta hedge. Delta-gamma hedging now reduces risk more efficiently and has a similar performance to the delta-vega hedging, implying that the importance of a convexity adjustment increases with longer hedging horizons. Under delta-vega hedging both the practitioner BSM and SM models outperform the Heston model for all moneyeness buckets and there is little change in the qualitative nature of our results, compared with those obtained with daily rebalancing.[18]

C Robustness Checks

Table 4 provides hedge error standard deviations over the two sub-samples. As expected, delta hedging results in a large reduction of risk, especially during the less volatile period (Panel A). Naturally, it was much more difficult to eliminate risk during the crisis and standard deviations of the hedging error were much greater in all categories. However, once an additional option is used in the hedge, the differences between the two periods reduce markedly. Clearly, gamma or vega hedging is particularly important during crisis periods, when such hedges improve upon the delta hedge by a factor of three or four, whereas in tranquil market periods the reduction in hedging error standard deviations is merely of the order of two or less.

[18] The hedging error is roughly proportional to the square root of the rebalancing frequency, so with weekly rebalancing hedging error increases by more than a factor of two. This merely reflects the increased uncertainty and is consistent with the reported increase from daily to weekly hedging errors reported in Bakshi, Cao, and Chen (1997).

Table 4: **Robustness.** This table reports the hedge error standard deviation over two subsamples. The models are abbreviated as TS (time-series calibration with $\lambda = 2$), Local (parameters from local calibration), Global (parameters from global calibration), ST (sticky tree), SM (sticky moneyness), SS (sticky strike) and UH (unhegded position).

	D	**MVD**	**DG**	**MVDG**	**DV**	**UH**	**D**	**MVD**	**DG**	**MVDG**	**DV**	**UH**
	Panel A: 1 January 2005 to 30 June 2008						Panel B: 1 July 2008 to 31 December 2008					
TS	7.42	5.04	5.03	3.54	3.37	35.75	21.54	18.05	20.06	5.73	5.46	71.48
Local	7.19	4.64	5.40	2.18	2.03	35.75	23.61	16.00	17.46	4.28	4.37	71.48
Global	7.01	5.15	4.81	2.66	2.30	35.75	21.93	17.20	16.71	4.79	4.62	71.48
ST	4.53		2.68		2.19	35.75	14.90		14.42		5.68	71.48
SM	7.15		4.76		2.07	35.75	23.73		23.82		3.91	71.48
SS	4.68		3.00		1.97	35.75	16.84		10.81		3.98	71.48

Finally, to investigate whether the Heston model is better for hedging long-term or short-term options we also divided the option data set along the maturity dimension. The rationale is that jumps in price or volatility may affect the short-term smile more than the long-term smile, in which case a pure stochastic volatility model could perform better for long-term options than it does for short-term contracts.[19] Yet, when hedging S&P 500 options, Bakshi, Cao, and Chen (1997) report that including jumps yields no discernible improvement. Our findings support this result, since the results in Table 4 are robust to the option's maturity. The detailed results disaggregated by option maturity are not included, for brevity, but are available from the authors upon request.

6 Conclusion

This paper has considered three main issues that have not been fully addressed in the previous option-hedging literature:

1. *What effect does the calibration procedure have on the hedging performance of a stochastic volatility model?* Calibration can affect the hedging results considerably. Dumas, Fleming, and Whaley (1998) find that over-fitting of a model can increase the variability of hedging errors, and frequent recalibration to option prices is not even consistent with most sochastic volatility models; yet daily recalibration of the Heston model to option prices clearly improves its hedging performance. This is true for all hedging strategies considered (delta, delta-gamma and delta-vega) and for both rebalancing frequencies (one day and one week). The results are also robust with respect to different sub-samples.

2. *Can minimum variance hedging improve upon simple adjustments to the Black-Scholes-Merton model?* The standard Heston model hedge ratios perform worse than any other model (except the delta-only hedge sticky-delta model and this is theoretically equiva-

[19] The stochastic volatility component – because of its diffusive nature – can not generate enough negative skewness and excess kurtosis in the short run. Price jumps on the other hand affect skewness and kurtosis even in the short run, see Bakshi, Cao, and Chen (1997).

lent to the standard Heston delta hedge). Minimum variance hedge ratio improve the performance very considerably, but they are still not able to outperform the simple smile-adjustments of the BSM hedges that are so popular with traders, at least when delta and delta-vega hedging. Only when delta-gamma hedging does it seem to be worthwhile using the Heston model.

3. *Is delta-vega hedging always better than delta-gamma hedging (or vice versa) or does it depend on the options being hedged, or the hedging model?* Overall, for a daily rebalancing strategy, it appears to be more important to hedge volatility risk before considering second order price risk. Delta-vega definitely leads to a greater risk reduction than delta-gamma hedging when the hedge is rebalanced daily, but with weekly rebalancing the two strategies have a similar performance. In fact, the excellent performance of the simple smile-adjusted delta-vega hedges are a little surprising, because the BSM model assumes that the volatility is constant and therefore one would expect the Heston model to capture volatility risk more accurately.

It should be emphasized that these conclusions have been reached only on the basis of hedging FTSE 100 index options. However, they are obtained using an extremely long sample period, in fact it is the longest period ever used in a hedging study to date. Our conclusions are robust to the market regime, whether it be unusually volatile or tranquil, and are reasonably robust across moneyness categories, the only noteworthy exception being the very high strike options. These options are the least liquid and the most difficult to hedge. It is for these options that we find the greatest improvements from using options prices for the Heston model calibration; in the other categories the differences can sometimes be fairly small.

A MCMC algorithm

In Bayesian statistics, one aims at recovering the posterior distribution of the unknown variables given the observed data $p\left(\mathbf{\Phi}, V \mid Y\right)$. Learning about this distribution is achieved by updating prior beliefs with the information obtained from observing the data. Thus formally we have

$$p\left(\mathbf{\Phi}, V \mid Y\right) \propto p\left(Y \mid \mathbf{\Phi}, V\right) p\left(\mathbf{\Phi}, V\right), \tag{10}$$

where $p\left(Y \mid \mathbf{\Phi}, V\right)$ denotes the *likelihood* and $p\left(\mathbf{\Phi}, V\right)$ the *prior density*. However, even though Eqn (10) gives a simple formula for the distribution of unobservable parameters, obtaining the density in closed form is impossible for most practical applications. Instead, a sampling algorithm has to be applied to produce random draws from this distribution. Then by sampling many times, we recover sufficient information about the shape of the density function and related quantities such as their marginals.

In order to overcome the problem of sampling from complex multivariate distributions, one can iteratively draw from the so-called complete conditional densities. For the problem at hand, these densities are $p\left(\phi_i \mid \mathbf{\Phi}_{-i}, V, Y\right)$ and $p\left(V_t \mid \mathbf{\Phi}, V_{-t}, Y\right)$, where the notation $\mathbf{\Phi}_{-i}$ indicates the parameter vector without the i-th element (same notation applies to the variance V) and $\phi_i \in \mathbf{\Phi}$. Once these distributions are derived, they can be used in a recursive procedure, hence replacing the simulation of the multivariate posterior distribution by simulation of several lower dimensional (here univariate) distributions. This procedure is known as the Gibbs sampler. Loosely speaking, it produces a Markov chain with invariant distribution equal to the posterior. Thus the chain can be used to obtain samples from the posterior distribution once convergence of the chain is achieved.

For the MCMC implementation of the Heston model, we need to specify prior distributions of the parameters μ, κ, θ, ξ and ϱ. Since we follow common practice and implement our algorithm on daily percentage returns (thus we use returns scaled by a factor 100), we need to reflect this scaling in the choice of our prior distributions. Our goal is to induce little knowledge through the priors and put the emphasis on the information contained in the observed return time series. Furthermore to facilitate sampling from the complete conditional distributions we opt for prior distributions of the conjugate class. In particular we choose $\mu \sim \mathcal{N}(0, 0.1)$, $\kappa \sim \mathcal{TN}^{+}(0, 1)$ and $\theta \sim \mathcal{TN}^{+}(0, 1)$, where $\mathcal{N}$ denotes the normal distribution with mean m and standard deviation s and $\mathcal{TN}^{+}$ denotes the truncated normal distribution (with truncation to the positive real axis and the two parameters now reflect the mean and standard deviation of the underlying normal density). This choice entails very little information considering typical parameter values obtained in the literature. We transform the parameters (ξ, ϱ) to (ψ_1, ψ_2) with $\psi_1 = \xi\varrho$ and $\psi_2 = \xi^2\left(1 - \varrho^2\right)$. This is motivated by the observation that the transformed parameters can be easily sampled as they are linear regression parameters for the variance time series (with slope ψ_1 and heteroscedastic error term ψ_2) for which sampling is standard. We use the uninformative priors $\psi_1 \sim \mathcal{N}(0, 1)$ and $\psi_2 \sim \mathcal{IG}(0.05, 1)$ where $\mathcal{IG}(m, s)$ denotes an inverse gamma distribution with mean m and standard deviation s. Having sampled from ψ_1 and ψ_2 we can transform back to $\xi = \sqrt{\psi_1^2 + \psi_2}$ and $\varrho = \psi_1/\xi$. For more details on this transformation we refer to Jacquier, Polson, and Rossi (2004). Li, Wells, and Yu (2008) also provide exact formulae for the complete conditional distributions of the Heston model augmented with jumps.

Sampling the variance vector is less straightforward for two reasons. Firstly, block updating for the parameter vector is not available and one has to cycle through the variance vector one by one. And secondly, posterior distributions for the individual variances are non standard. By the Markov property and the Bayes formula we obtain (with appropriate adjustments for the first and last variance)

$$p\ (V_t \,|\, V_{-t}, Y, \mathbf{\Phi}) \propto p\ (Y_{t+1} \,|\, V_t, \mathbf{\Phi})\, p\ (V_{t+1} \,|\, V_t, Y_{t+1}, \mathbf{\Phi})\, p\ (V_t \,|\, V_{t-1}, Y_t, \mathbf{\Phi})\,,$$

which is rather involved as the variance parameter enters in several place on the right hand side. We have tested two updating algorithms, the random walk Metropolis and the adaptive rejection Metropolis sampling (ARMS) algorithm of Gilks, Best, and Tan (1995). We found that both produce accurate results for this model, indeed estimated parameters for the FTSE index where virtually the same for both algorithms. We ended up reporting results from the ARMS algorithm.

References

Alexander, C. and Nogueira, L. M. (2007). Model-free hedge ratios and scale-invariant models. *Journal of Banking & Finance*, **31**(6), 1839–1861.

Alexander, C., Kaeck, A., and Nogueira, L. M. (2009). Model adjusted hedge ratios. *Journal of Futures Markets*, **29**(11), 1021–1049.

Bakshi, G., Cao, C., and Chen, Z. (1997). Empirical performance of alternative option pricing models. *Journal of Finance*, **52**(5), 2003–49.

Bakshi, G., Cao, C., and Chen, Z. (2000). Pricing and hedging long-term options. *Journal of Econometrics*, **94**(1-2), 277–318.

Bates, D. S. (2000). Post-'87 crash fears in the S&P 500 futures option market. *Journal of Econometrics*, **94**(1-2), 181–238.

Bates, D. S. (2005). Hedging the smirk. *Financial Research Letters*, **2**(4), 195–200.

Black, F. (1976). The pricing of commodity contracts. *Journal of Financial Economics*, **3**(1-2), 167–179.

Black, F. and Scholes, M. S. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, **81**(3), 637–54.

Bliss, R. R. and Panigirtzoglou, N. (2004). Option-implied risk aversion estimates. *Journal of Finance*, **59**(1), 407–446.

Branger, N., Schlag, C., Schneider, E., and Seeger, N. (2008). Hedging under model mis-specification: Which risk factors should you not forget? *Working Paper*.

Britten-Jones, M. and Neuberger, A. (2000). Option prices, implied price processes, and stochastic volatility. *Journal of Finance*, **55**(2), 839–866.

Broadie, M., Chernov, M., and Johannes, M. (2007). Model specification and risk premia: Evidence from futures options. *Journal of Finance*, **62**(3), 1453–1490.

Carr, P. and Madan, D. (1999). Option valuation using the fast fourier transform. *Journal of Computational Finance*, **2**(4), 61–73.

Chernov, M. and Ghysels, E. (2000). A study towards a unified approach to the joint estimation of objective and risk neutral measures for the purpose of options valuation. *Journal of Financial Economics*, **56**(3), 407–458.

Christoffersen, P. and Jacobs, K. (2004). The importance of the loss function in option valuation. *Journal of Financial Economics*, **72**(2), 291–318.

Christoffersen, P., Jacobs, K., and Mimouni, K. (2010). Volatility dynamics for the S&P 500: Evidence from realized volatility, daily returns, and option prices. *Review of Financial Studies, forthcoming*.

Coleman, T., Kim, Y., Li, Y., and Verma, A. (2001). Dynamic hedging with a deterministic local volatility function model. *Journal of Risk*, **4**(1), 63–89.

Derman, E. and Kani, I. (1994). Riding on a smile. *Risk*, **7**(2), 32–29.

Derman, E., Kani, I., and Zou, J. Z. (1996). The local volatility surface: Unlocking the information in index option prices. *Financial Analysts Journal*, **52**(4), 25–36.

Dotsis, G., Psychoyios, D., and Skiadopoulos, G. (2007). An empirical comparison of continuous-time models of implied volatility indices. *Journal of Banking & Finance*, **31**(12), 3584–3603.

Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, **68**(6), 1343–1376.

Dumas, B., Fleming, J., and Whaley, R. E. (1998). Implied volatility functions: Empirical tests. *Journal of Finance*, **53**(6), 2059–2106.

Dupire, B. (1994). Pricing with a smile. *Risk*, **7**(1), 18–20.

Engelmann, B., Fengler, M. R., and Schwendner, P. (2006). Better than its reputation: An empirical hedging analysis of the local volatility model for barrier options. *Unpublished Working Paper*.

Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *Journal of Finance*, **59**(3), 1367–1404.

Eraker, B., Johannes, M., and Polson, N. (2003). The impact of jumps in volatility and returns. *Journal of Finance*, **53**(3), 1269–1300.

Gilks, W. R., Best, N. G., and Tan, K. K. (1995). Adaptive rejection metropolis sampling within gibbs sampling. *Applied Statistics*, **44**(4), 455–472.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, **6**(2), 327–43.

Jacquier, E., Polson, N. G., and Rossi, P. E. (1994). Bayesian analysis of stochastic volatility models. *Journal of Business & Economic Statistics*, **12**(4), 371–89.

Jacquier, E., Polson, N. G., and Rossi, P. E. (2004). Bayesian analysis of stochastic volatility models with fat-tails and correlated errors. *Journal of Econometrics*, **122**(1), 185–212.

Johannes, M. and Polson, N. (2006). MCMC methods for financial econometrics. In *Yacine Aït-Sahalia and Lars Hansen (eds.), Handbook of Financial Econometrics*. New York: Elsevier.

Jones, C. S. (2003). The dynamics of stochastic volatility: evidence from underlying and options markets. *Journal of Econometrics*, **116**, 181–224.

Kim, I. J. and Kim, S. (2004). Empirical comparison of alternative stochastic volatility option pricing models: Evidence from korean KOSPI 200 index options market. *Pacific-Basin Finance Journal*, **12**(2), 117–142.

Kim, I. J. and Kim, S. (2005). Is it important to consider the jump component for pricing and hedging short-term options? *Journal of Futures Markets*, **25**(10), 989 – 1009.

Kim, S. (2009). The performance of traders' rules in options market. *Journal of Futures Markets*, **29**(11), 999–1020.

Lewis, A. L. (2000). *Option Valuation under Stochastic Volatility*. Finance Press.

Li, H., Wells, M. T., and Yu, C. L. (2008). A bayesian analysis of return dynamics with lévy jumps. *Review of Financial Studies*, **21**(5), 2345–2378.

Merton, R. C. (1973). Theory of rational option pricing. *Bell Journal of Economics*, **4**(1), 141–183.

Pan, J. (2002). The jump-risk premia implicit in options: evidence from an integrated time-series study. *Journal of Financial Economics*, **63**, 3–50.

Poulsen, R., Schenk-Hoppé, K. R., and Ewald, C.-O. (2009). Risk minimization in stochastic volatility models: Model risk and empirical performance. *Quantitative Finance*, **9**, 693–704.

Psychoyios, D. and Skiadopoulos, G. (2006). Volatility options: hedging effectiveness, pricing, and model error. *Journal of Futures Markets*, **26**(1), 1–31.

Rubinstein, M. (1994). Implied binomial trees. *Journal of Finance*, **49**(3), 771–818.

Vähämaa, S. (2004). Delta hedging with the smile. *Financial Markets and Portfolio Management*, **18**(3), 241–255.